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Optimal pension fund management in a jump–diffusion environment: Theoretical and empirical studies

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ABSTRACT

This paper considers a theoretical and an empirical study of an optimal pension fund in an inflation environment in which the consumption–portfolio selection problem of an investor who faces both diffusion and jump risks was analyzed. Since the time horizon of a pension fund management is relatively long, we put into consideration four background risks which include inflation, interest rate, investment and income risks. A pension plan member (PPM) is expected to contribute continuously a time-consistent proportion of his income into the scheme. These contributions are invested into a market that is characterized by multiple risk-free assets (which include riskless bonds and bank deposit accounts), stocks and index bonds. The risky assets (stocks and index bonds), interest rates and income process are assumed to follow a jump–diffusion process. Real wealth for the plan member is considered. The resulting Hamilton–Jacobi–Bellman equation was solved using dynamic programming approach. From which, the optimal consumption and optimal investments with jump risks were obtained. Empirical data were collected from Nigeria Stock Exchange; National Bureau of Statistics, Nigeria; and Bursary Department, University of Benin, Nigeria. The analyses of the data were carried out using SPSS package in order to obtain the needed information for the parameters in our derived models. The resulting models for optimal portfolio and consumption were solved using MatLab and Mathematica software. We found that inflation, interest rate and income risks have significant influence on the investor's portfolio values in the risky assets. We also found that as the risk averse coefficient increases, consumption increases and vice versa. Furthermore, we found that an increase in income can lead to an increase in the portfolio risks and vice versa.

1. Introduction

This paper focuses on solving problems faced by countries in Africa and many other countries of the World that are adopting defined contribution pension scheme. Today, there are general increase in prices of goods and services, lack of price control, varying interest rates by financial institutions, price processes experiencing jump–diffusions and so many other problems. All of these will go a long way to affect the plan member savings in pension scheme. We intend to provide both theoretical and empirical analyses of these problems as it relates to pension plan.

In Nigeria for instance, consumer prices in recent time are experiencing continuous jumps from January 2016 to June 2016. In January 2016, the inflation rate was 9.6%, in February, inflation rate was 11.4%, March, 12.8%, April, 13.7% and as at June 15, inflation rate has risen to 15.6%. It is the highest inflation rate in over 6 years, as cost of items such as food, housing, utilities and transport surged as a result of 67 percent increase in the prices of gasoline, see [1]. Fig. 1 shows bar chart of inflation rate in Nigeria from June 2015 to June 2016. At the moment, Nigeria is ranked number 7 highest inflation

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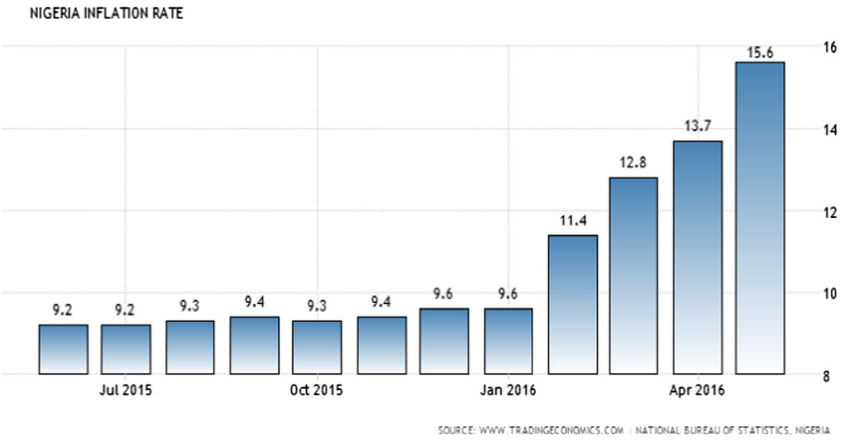

Fig. 1. Nigeria Inflation Rate in past one year.

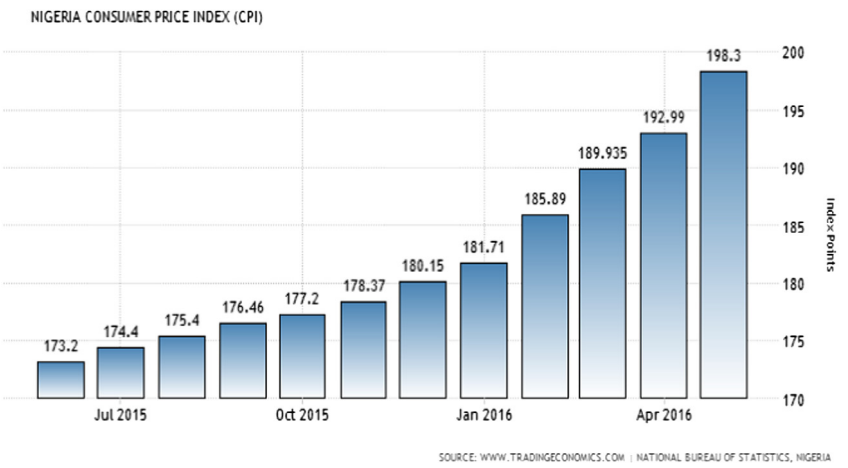

Fig. 2. Nigeria Consumer Price Index from June 2015 to June 2016 as by the National Bureau of Statistics, Nigeria as at June of 2016.

rate in Africa and number 12 in the World. The highest in the World is South Sudan with an inflation rate of about 290.30%, followed by Venezuela with 180.90%, according to the National Bureau of Statistics, Nigeria as at June of 2016. In Nigeria, the Consumer Price Index (CPI) measures changes in the prices paid by consumers in Nigeria for a basket of goods and services over time in day-to-day living. CPI in Nigeria increased to 198.30 Index Points as at June, 2016 from 192.99 Index Points in April of 2016. CPI in Nigeria averaged 77.79 Index Points from 1995 to 2016, reaching an all time high of 198.30 Index Points in June of 2016 and a record low of 14.36 Index Points in January of 1995 as reported by the National Bureau of Statistics, Nigeria in June 2016. Fig. 2 shows the bar chart of CPI as reported by the National Bureau of Statistics, Nigeria as at June of 2016. Since inflation rate and stocks are generally experiencing jumps, we must model and assume that our CPI, interest rate, index bonds and stocks follow jump–diffusion processes. This paper aims at providing both theoretical and empirical studies of pension fund management in which the environment is made up of general jump in CPI, index bonds, interest rate, income and stocks prices.

We now give the highlights of our research work as follows:

1. time-dependent contribution rate is considered
2. jump–diffusion interest rate and jump–diffusion income process are considered
3. consumer price index is allowed to follow jump–diffusion process
4. multiple riskless assets, multiple index bonds and multiple stocks are considered
5. optimal investment and optimal consumption are obtained
6. empirical data were used to analyze the resulting models
7. real wealth and consumption plan for a PPM are considered
8. four background risks: interest rate, inflation, investment and income risks are considered