Journal of Computational and Applied Mathematics 330 (2018) 318–329

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Detecting buried wave-penetrable scatterers in a two-layered medium

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ARTICLE INFO

Article history:
Received 7 February 2017
Received in revised form 24 July 2017

Keywords:
Inverse scattering
Two-layered medium
Buried scatterer
Uniqueness
Multilevel sampling method

ABSTRACT

In a two-layered medium, we prove that a buried inhomogeneous scatterer is uniquely determined from the wave field data measured in the upper half-space with respect to many incident point sources. Moreover, we extend the multilevel sampling method in Liu and Zou (2013) to numerically reconstruct the buried scatterer applying only few incident fields and partial scattered data. The extended recovery scheme only involves matrix–vector operations and does not need to solve any large-scale ill-posed linear systems or any optimization process. It is feasible to deal with the scatterers of different features and easy to implement, highly tolerant to noise and computationally quite cheap. We can regard it as an effective yet simple computational method to provide a reliable initial guess for the implementation in existing more accurate and refined optimization-type reconstruction algorithms.

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1. Introduction

In this paper, we consider the reconstruction of wave penetrable scatterers buried in a two-layered medium; see Fig. 1 for the geometrical demonstration. This work can find wide applications in landmines detection in the battlefield, anti-submarine locating and underground mineral prospection.

Assume that the whole space $\mathbb{R}^n$, $n = 2, 3$, is divided by an unbounded interface Γ into two half spaces, denoted by the upper one $\mathbb{R}^n_+$ and the lower one $\mathbb{R}^n_-$. The interface Γ is *a priori* known flat plane; see Γ in Fig. 1. The mediums in $\mathbb{R}^n_+$ and $\mathbb{R}^n_-$ are assumed to be homogeneous with two different wavenumber k_+ and k_-, respectively. It is further supposed that some inhomogeneous scatterers denoted by Ω are embedded in the lower half-space, which is described by the contrast function $q(x) \in L^\infty(\mathbb{R}^n)$ satisfying that q vanishes outside Ω, see Fig. 1.

Given $(\Omega, \Gamma, k_\pm, q)$, consider the scattering by a certain incident wave induced in the upper-half space. The forward problem is to find the distribution of the perturbed wave field due to Ω. And the inverse problem we are concerned with is to recover the contrast function q or its support Ω from the knowledge of scattered wave fields measured in the upper-half space. It is well-known that two typical features of this class of problems are highly nonlinear and severely ill-posed. Compared with the case of a homogeneous background medium, only partial scattered wave data are available in the two-layered media, therefore, it is more challenging.

To the authors' knowledge, very few uniqueness results are available in the literature for the recovery of a buried inhomogeneous medium or a wave-penetrable scatterer in a two-layered medium, especially in the two-dimensional case.

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http://dx.doi.org/10.1016/j.cam.2017.08.021
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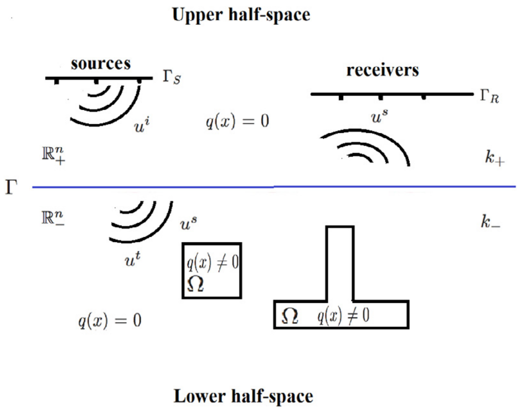

Fig. 1. Geometrical model of the wave-penetrable scatterers detection in the two-layered medium.

A related inverse problem was studied in [1], where a uniqueness result was obtained for a buried impenetrable obstacle in inverse electromagnetic scattering problem. It is noted that a new method [2] was recently proposed in order to obtain the uniqueness of a wave-penetrable object from the far-field data. It was shown in [2] that one important feature of the method is to construct a 'local' well-posed interior transmission problem by the solution of scattering problem and can thus lead to a contradiction. Motivated by this idea, we shall extend the work of [2] to the case of the two-layered background media with the near-field data, corresponding to many point sources.

Compared to few uniqueness results, several numerical schemes have been developed for the estimation of buried scatterer in literatures [3–10]. One popular and effective type of methods is the so-called sampling method which is to construct different index functions that can provide significantly differences when the sampling point lies in the interior and exterior of the scatterer. The main geometrical feature of the scatterer could be thus identified by applying such indicators which leads to a fast imaging algorithm. We refer to some related works in the homogeneous background medium, e.g., the MUSIC-type methods [11], the linear sampling method (LSM) [12], the direct sampling method (DSM) [13,14], the contrast source inversion method (CSI) [15], the factorization method [11], the methods with multifrequency data [16,17], etc. In order to carry out any of these schemes for the reconstruction of unknown multiple scatterers, the initial important step is to efficiently estimate some domains that contain all the scatterers. Without such an effective step, a computational region of a size that is much larger than the actual sizes of all scatterers would be considered in the recovering procedure. And a much larger computational region results usually in a huge additional computational burden for the entire numerical reconstruction process, considering the severe ill-posedness and strong nonlinearity of inverse medium scattering problems. Therefore, it is of great significance to provide appropriate initial computational regions that contain all the scattering obstacles for the inverse medium scattering problems. Recently, several effective techniques [13,18,19] are proposed to tackle this problem in a homogeneous space, all these methods are less expensive computationally and easy to implement numerically. More notably, they are entirely direct which means they do NOT need any optimization process, matrix inversions or to solve any large-scale ill-posed linear systems.

In the two-layered medium, the method in [20] is only extended to detect the wave impenetrable scatterers by the far-field data. In this work, we would like to extend the multilevel sampling method (MSM) [19] to provide the initial computational domain for the reconstruction of wave-penetrable scattering objects using the near-field data in the two-layered medium. With an effective initial position of each scatterer, we may then implement any existing efficient but computationally more demanding techniques, i.e., the methods in [3,15–17], for further refinement of the estimated physical features of each scatterer as well as for the recovery of the contrast profiles of different media. This extended algorithm is an iterative one, and only three matrix–vector multiplications are needed at each iteration, without any matrix inversion or solutions of linear systems involved. And the computational sampling region shrinks quickly during the iterative process. Therefore, the extended method is easy to implement and advantage in computational complexity for both 2D and 3D cases. Most interestingly, the extended algorithm is able to separate all disjoint inhomogeneous medium scatterers quickly, usually in a few iterations, then refine its estimation successively and finally provide a reliable approximate region for each separated obstacle. In addition, the newly proposed method is able to recover multiscale scatterers as well. It is worth to mention that another novel MSM has already been successfully applied to detect sources in a three-layered waveguide model [21].

The paper is organized as follows. In Section 2, the direct scattering problem of the two-layered medium are stated, along with some useful notation, identities and properties. Section 3 describes the uniqueness for the inverse problem and the extended multilevel sampling method using near-field partial scattered data, and Section 4 provides extensive numerical examples to evaluate the effectiveness of the extended multilevel sampling method. Finally, some concluding remarks are presented in Section 5.

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