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A SIMPLE FINITE ELEMENT METHOD FOR LINEAR HYPERBOLIC PROBLEMS

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Abstract. In this paper, we introduce a simple finite element method for solving first order hyperbolic equations with easy implementation and analysis. This new method, with a symmetric, positive definite system, is designed to use discontinuous approximations on finite element partitions consisting of arbitrary shape of polygons/polyhedra. Error estimate is established. Extensive numerical examples are tested that demonstrate the robustness and flexibility of the method.

Key words. finite element methods, hyperbolic equations

AMS subject classifications. Primary, 65N30, 65N15, 65N12, 74N20; Secondary, 35B45, 35J50, 35J35

1. Introduction. We consider the model problem that seeks an unknown function $u = u(\mathbf{x})$ satisfying

$$\begin{aligned} \nabla\cdot(\boldsymbol{\beta} u) =& f, \qquad \text{in} \quad \Omega, \\ u =& g, \qquad \text{on} \quad \Gamma_-, \end{aligned} \tag{1.1}$$

where $\boldsymbol{\beta}$ is a flow field, Γ_- is the inflow boundary defined as follows

$$\Gamma_- = \{\mathbf{x} \in \partial\Omega,\ \boldsymbol{\beta}(\mathbf{x})\cdot\mathbf{n}(\mathbf{x}) < 0\},$$

and $\Gamma_+ = \partial\Omega \setminus \Gamma_-$. We assume that the model problem (1.1) has a unique solution. This first order hyperbolic equation has many applications in science and engineering.

Linear hyperbolic partial differential equations may have discontinuous solutions when the boundary data is discontinuous. The nature of this type of problems makes it difficult to develop numerical methods for resolving discontinuities without introducing spurious oscillations. Numerical solutions of the first order hyperbolic equations have been investigated intensively and many different numerical schemes have been developed such as SUPG and continuous/discontinuous Galerkin methods [6, 9] and references therein.

The goal of this paper is to develop a simple and flexible finite element scheme to solve the hyperbolic equation (1.1). Our method can be derived by using discontinuous

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