Accepted Manuscript

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PII: S0377-0427(17)30450-8
DOI: http://dx.doi.org/10.1016/j.cam.2017.09.019
Reference: CAM 11299

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 28 April 2015
Revised date : 1 August 2017

Weak Galerkin finite element method for Biot's consolidation problem * †‡§

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Abstract

In this paper, a fully discrete weak Galerkin (WG) finite element method is proposed to solve Biot's consolidation problem, where weakly defined gradient and divergence operators over discontinuous functions are introduced. $\mathbb{P}_l$-$\mathbb{P}_l$ $(l \geq 1)$ finite element combination is used for the displacement and pressure approximations in the interior of the elements, and $\mathbb{P}_l$-$\mathbb{P}_{l-1}$ combination for the corresponding trace approximations on the interfaces of the finite element partition. The existence and uniqueness of the discrete linear system at each time step is derived, and error estimates for the approximation of displacement and pressure are obtained. Numerical experiments confirm the theoretical results and show that the proposed WG method is capable of overcoming pressure oscillations.

1 Introduction

Let Ω be a convex polyhedral domain in $\mathbb{R}^d (d = 2, 3)$ with boundary $\Gamma = \Gamma_D \bigcup \Gamma_N = \Gamma_{\tilde{D}} \bigcup \Gamma_{\tilde{N}}$, where meas$(\Gamma_D) > 0$ and meas$(\Gamma_{\tilde{D}}) > 0$. For $t \in (0, T]$, we consider Biot's consolidation problem [5]

$$-\nabla \cdot (2\mu\varepsilon(\mathbf{u}) + \lambda \nabla \cdot \mathbf{u}\mathbf{I}) + \nabla p = \mathbf{f} \quad \text{in} \quad \Omega \times (0, T], \tag{1.1}$$

$$\nabla \cdot D_t \mathbf{u} - \frac{\kappa}{\eta_{\mathbf{f}}} \Delta p = g \quad \text{in} \quad \Omega \times (0, T], \tag{1.2}$$

with the boundary conditions

$$\mathbf{u} = \mathbf{0} \text{ on } \Gamma_D, \quad (2\mu\varepsilon(\mathbf{u}) + \lambda \nabla \cdot \mathbf{u}\mathbf{I} - p\mathbf{I})\mathbf{n} = \boldsymbol{\beta} \text{ on } \Gamma_N, \tag{1.3}$$

$$p = 0 \text{ on } \Gamma_{\tilde{D}}, \quad \frac{\kappa}{\eta_{\mathbf{f}}} \nabla p \cdot \mathbf{n} = \gamma \text{ on } \Gamma_{\tilde{N}} \tag{1.4}$$

and the initial condition

$$\nabla \cdot \mathbf{u}(0) = 0 \quad \text{in} \quad \Omega. \tag{1.5}$$

*This work was supported by National Natural Science Foundation of China (11271390), Scientific Research of SiChuan Provincial Education Department(15ZA0149), and Major Research Plan of National Natural Science Foundation of China (91430105) .

†2000Mathematics Subject Classification. Primary 65N12, 65N30.

‡Key Words. finite element method, Biot's consolidation problem, weak Galerkin method

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