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Numerical approximation of a Tikhonov type regularizer by a discretized frozen steepest descent method

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Abstract

We present a frozen regularized steepest descent method and its finite dimensional realization for obtaining an approximate solution for the nonlinear ill-posed operator equation $F(x) = y$. The proposed method is a modified form of the method considered by Argyros et al. (2014). The balancing principle considered by Pereverzev and Schock (2005) is used for choosing the regularization parameter. The error estimate is derived under a general source condition and is of optimal order. Numerical example provided proves the efficiency of the proposed method.

MSC: 47A52, 65J15
Keywords: Nonlinear ill-posed problem; Steepest descent method; balancing principle.

1 Introduction

Inverse problems arise in many practical applications, such as inverse scattering problem, tomographic, parameter identification in partial differential equations (see [5,9,13]). They can be modeled as an operator equation

$$F(x) = y, \tag{1.1}$$

where $F : D(F) \subseteq X \to Y$ is a nonlinear Fréchet differentiable operator between the Hilbert spaces X and Y. Throughout this study, $D(F)$, $\langle .,. \rangle$ and $\|.\|$, respectively stand for the domain of F, innerproduct and norm which can always be identified from the context in which they appear. Fréchet derivative of F is denoted by $F'(.)$ and its adjoint by $F'(.)^*$. Further we assume that equation (1.1) has a solution $\hat{x}$, which is not depending continuously on the right-hand side data y. The problems in which the solution $\hat{x}$ is not depending continuously on the right hand data are called ill-posed problems. It is a common practice to use iterative methods or iterative regularization methods for approximating $\hat{x}$. For example, Landweber method ([10,23]), Leveberg-Marquardt method ([11]), Gauss-Newton ([3,4]), Conjugate Gradient ([12]), Newton-like methods ([6,16]), TIGRA (Tihkonov-gradient method) ([22]).

It is assumed further that we have only approximate data $y^\delta \in Y$ with

$$\|y - y^\delta\| \leq \delta.$$

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