Journal of Computational and Applied Mathematics 330 (2018) 586–604

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

An extended nonmonotone line search technique for large-scale unconstrained optimization☆

Shuai Huang [a], Zhong Wan [a,*], Jing Zhang [b,*]

[a] School of Mathematics and Statistics, Central South University, Hunan, Changsha, China
[b] Experimental Teaching Center, Guangdong University of Foreign Studies, Guangdong Guangzhou, China

HIGHLIGHTS

- New nonmonotone line search is proposed to integrate advantages of the existing ones.
- Theory of global and local convergence has been established.
- Numerical results show its efficiency for solving large-scale optimization problems.

ARTICLE INFO

Article history:
Received 7 January 2017
Received in revised form 5 July 2017

MSC 2010:
90C30
62K05
68T37

Keywords:
Algorithm
Large-scale optimization
Nonmonotone line search
Global convergence

ABSTRACT

In this paper, an extended nonmonotone line search is proposed to improve the efficiency of the existing line searches. This line search is first proved to be an extension of the classical line search rules. On the one hand, under mild assumptions, global convergence and R-linear convergence are established for the new line search rule. On the other hand, by numerical experiments, it is shown that the line search can integrate the advantages of the existing methods in searching for a suitable step-size. Combined with the spectral step-size, a class of spectral gradient algorithms are developed and employed to solve a large number of benchmark test problems from CUTEst. Numerical results show that the new line search is promising in solving large-scale optimization problems, and outperforms the other similar ones as it is combined with a spectral gradient method.

© 2017 Elsevier B.V. All rights reserved.

1. Introduction

Consider the following unconstrained optimization problem:

$$\min_{x\in\mathbb{R}^n} f(x), \tag{1.1}$$

where $f:\mathbb{R}^n\to\mathbb{R}$ is a continuously differentiable function and its gradient $g:\mathbb{R}^n\to\mathbb{R}^n$ is available. To solve (1.1), iterative algorithms are often developed to generate a sequence of approximate solutions $\{x_k\}$, from an initial given point x_0. The relation between two successive points x_k and x_{k+1} is given by

$$x_{k+1} = x_k + \alpha_k d_k,$$

☆ This research is supported by the National Natural Science Foundation of China (Grant No. 71671109), the Hunan Provincial Innovation Foundation For Postgraduate (CX2015B038), the Natural Science Foundation of Guangdong, China (2016A030310105) and the Scientific Research Fund of Hunan Provincial Education Department, China (15B022).

* Corresponding authors.
E-mail addresses: wanmath@csu.edu.cn (Z. Wan), zhangjing325@126.com (J. Zhang).

http://dx.doi.org/10.1016/j.cam.2017.09.026
0377-0427/© 2017 Elsevier B.V. All rights reserved.

where d_k is a search direction at the kth iteration which is often a descent search direction, and α_k is a step-size along d_k, often obtained by some line search rule.

A well-known line search is the Armijo line search, which chooses α_k to be the largest component in the set $\{\alpha_k \mid \alpha_k = \bar{\alpha}_k \rho_1^l, l \in \{0, 1, 2, \ldots, \}\}$ such that

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k, \tag{1.2}$$

where $\bar{\alpha}_k$ is a trial step-size and is often set to be 1, $g_k = g(x_k)$, and $\rho_1, \delta \in (0, 1)$ are given constants. Due to lower computational cost, many variants of (1.2) have been presented in the literature (see, for example, [1–5]). As d_k is a descent search direction, the sequence $\{f(x_k)\}$ is monotonically decreasing [6]. From this viewpoint, the Armijo line search and the majority of its variants belong to strategies of monotone line search.

However, if the iterative sequence $\{x_k\}$ is trapped near a narrow curved valley, any monotone line search would be inefficient owing to generation of very short or zigzagging steps (see [7–9]). To overcome this drawback, nonmonotone line search is often adopted, which allows the values of objective function to increase occasionally (see [7–19]). It is noted that the nonmonotone line search may improve the likelihood of finding a global optimal solution and the rate of convergence (see [9,11]).

The first nonmonotone line search was proposed in [7]. It finds a step-size as follows: (1) Fix a nonnegative integer M; (2) Choose two constants $\delta, \rho_1 \in (0, 1)$; (3) Take an initial trial step $\bar{\alpha}_0 > 0$; (4) Find a step-size $\alpha_k = \bar{\alpha}_k \rho_1^{h_k}$ satisfying

$$f(x_k + \alpha_k d_k) \leq \max_{0 \leq j \leq m(k)} f(x_{k-j}) + \delta \alpha_k g_k^T d_k, \tag{1.3}$$

where $0 \leq m(k) \leq \min\{m(k-1)+1, M\}$, $m(0) = 0$, $\bar{\alpha}_k$ is a prior trial step-size, and h_k is the first nonnegative integer such that (1.3) is satisfied. Owing to the maximum function, the value of M seriously affects the numerical performance of algorithms (see [7,9,15]).

Different from (1.3), another nonmonotone line search rule was presented by Zhang and Hager in [18]. Specifically, with an initial point x_0, compute $C_0 = f(x_0)$. Set $Q_0 = 1$. Choose $0 \leq \eta_{min} \leq \eta_{max} \leq 1 < \rho, \delta \in (0, 1)$ and $\mu > 0$. For $k \geq 1$, choose $\eta_k \in [\eta_{min}, \eta_{max}]$ and a step-size $\alpha_k = \bar{\alpha}_k \rho^{h_k}$, where $\alpha_k \leq \mu$, $\bar{\alpha}_k$ is a trial step, and h_k is the largest integer which satisfies the following inequality:

$$f(x_k + \alpha_k d_k) \leq C_k + \delta \alpha_k g_k^T d_k, \tag{1.4}$$

where

$$Q_{k+1} = \eta_k Q_k + 1, \quad C_{k+1} = \frac{\eta_k Q_k C_k + f(x_k + \alpha_k d_k)}{Q_{k+1}}. \tag{1.5}$$

It is clear that C_{k+1} in (1.5) is a convex combination of all the function values $f(x_0), f(x_1), \ldots, f(x_{k+1})$. In (1.4), similar to M in (1.3), the choice of η_k plays a critical role in controlling the degree of nonmonotonicity (see [16,18]).

In more than two decades, a great deal of research focuses on studying possible variants of the two nonmonotone line search rules (1.3) and (1.4) (see [10,20,21]). For example, an extension of (1.4) was proposed very recently in [21]. Specifically, let $0 \leq \eta_{min} \leq \eta_{max} < 1 < \rho$, $\delta_{max} < 1$, $0 < \delta_{min} < (1 - \eta_{max})\delta_{max}$, and $\mu > 0$. For $k \geq 1$, choose $\eta_k \in [\eta_{min}, \eta_{max}]$, $\delta_k \in [\delta_{min}, \frac{\delta_{max}}{Q_{k+1}}]$. Then, the line search method (1.4) is replaced by

$$C_{k+1} = \frac{\eta_k Q_k C_k + f(x_k + \alpha_k d_k)}{Q_{k+1}} \leq C_k + \delta_k \alpha_k g_k^T d_k. \tag{1.6}$$

As proved in [21], the new line search (1.6) possesses a number of better properties than (1.4). Another focus on nonmonotone line searches is to apply these line search rules to solve the problems from other research areas (see [22,23]).

Different from the existing results, this paper is intended to propose a new rule of nonmonotone line search such that it has more advantages than the others. Our basic ideas can be simply stated as follows.

- The new rule should integrate the advantages of all the existing nonmonotone line search methods (1.3), (1.4) and (1.6). Actually, we will prove that any one among the existing methods can be seen as a special case of our line search rule. As a result, one of main contributions in this paper is to overcome difficulties caused by establishing convergence theory under more generic line search rule.
- As done for all the other similar methods, it should be verified that the new line search can improve numerical performance of algorithms. Actually, we will test our method by solving a large number of large-scale benchmark test problems from CUTEst. The numerical results indicate that: (1) The adjustability of the algorithmic parameters in the new line search can improve its adaptability to solve more optimization problems, compared with the existing methods; (2) Its numerical efficiency outperforms that of the similar methods.

The rest of this paper is organized as follows. In Section 2, a new strategy of nonmonotone line search is proposed and some properties are analyzed. Section 3 is devoted to development of an algorithm based on the new line search. Convergence theory of the algorithm is also established. In Section 4, numerical performance of the developed algorithm is reported. Some conclusions are made in Section 5.

Download English Version:

https://daneshyari.com/en/article/5776077

Download Persian Version:

https://daneshyari.com/article/5776077

Daneshyari.com