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Symmetric second derivative integration methods

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Abstract

The purpose of this paper is the derivation of symmetric second derivative general linear methods (SGLMs) for general time-reversible differential equations. For this purpose, we find algebraic conditions on the coefficients matrices of the methods which are sufficient for the method to be symmetric. Some symmetric methods of order 4 are constructed and implemented on the famous reversible problems. Numerical experiments show that the constructed symmetric SGLMs approximately conserve the invariants of motion over long time intervals for reversible Hamiltonian systems.

Keywords: Hamiltonian systems, General linear methods, Second derivative methods, Symmetric methods, G-symplecticity, Reversible problems

1. Introduction

The construction of numerical schemes for solving ordinary differential equations (ODEs) which preserve some qualitative geometrical properties of the flow of the differential equation is a relatively new area of numerical analysis. The symmetric property has been recognized as an important factor for the good long-time behaviour of a numerical method applied to time-reversible differential equations, as it is the case of conservative mechanical systems including Hamiltonian systems. Symmetric methods play a central role in the geometric integration of differential equations which a theoretical explanation of the good long-time behaviour of these methods is given in [20].

The differential equation $y' = f(y)$ is called ρ-reversible if

$$f(\rho y) = -\rho f(y) \quad \text{for all } y, \tag{1}$$

with ρ as an invertible linear transformation in the phase space [20]. This implies that the flow of the system, denoted by φ_t, satisfies $\rho \circ \varphi_t = \varphi_t^{-1} \circ \rho$. A Hamiltonian system with the Hamiltonian function satisfying $H(p,q) = H(-p,q)$ is an example of reversible problems with the reflection $\rho(p,q) = (-p,q)$.

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