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Computationally enhanced projection methods for symmetric Sylvester and Lyapunov matrix equations☆

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Abstract

In the numerical treatment of large-scale Sylvester and Lyapunov equations, projection methods require solving a reduced problem to check convergence. As the approximation space expands, this solution takes an increasing portion of the overall computational effort. When data are symmetric, we show that the Frobenius norm of the residual matrix can be computed at significantly lower cost than with available methods, without explicitly solving the reduced problem. For certain classes of problems, the new residual norm expression combined with a memory-reducing device make classical Krylov strategies competitive with respect to more recent projection methods. Numerical experiments illustrate the effectiveness of the new implementation for standard and extended Krylov subspace methods.

Keywords: Sylvester equation, Lyapunov equation, projection methods, Krylov subspaces.
2000 MSC: 47J20, 65F30, 49M99, 49N35, 93B52

1. Introduction

Consider the Sylvester matrix equation

$$AX + XB + C_1C_2^T = 0, \quad A \in \mathbb{R}^{n_1\times n_1},\ B \in \mathbb{R}^{n_2\times n_2},\ C_1 \in \mathbb{R}^{n_1\times s}, C_2 \in \mathbb{R}^{n_2\times s} \quad (1)$$

where A, B are very large and sparse, symmetric negative definite matrices, while $C_1, C_2 \neq 0$ are tall, that is $s \ll n_1, n_2$. Under these hypotheses, there exists a unique solution matrix X. This kind of matrix equation arises in many applications, from the analysis of continuous-time linear dynamical systems to

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