Accepted Manuscript

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PII: S0377-0427(17)30205-4
DOI: http://dx.doi.org/10.1016/j.cam.2017.04.034
Reference: CAM 11114

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 23 December 2016

Please cite this article as: C. Lizama, J. Pereira, E. Toon, On the exponential stability of Samuelson model on some classes of times scales, *Journal of Computational and Applied Mathematics* (2017), http://dx.doi.org/10.1016/j.cam.2017.04.034

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Abstract

We prove the remarkable property that the exponential stability of the Samuelson model is invariant under the times scales $\mathbb{R}$ and $h\mathbb{Z}$ whenever $0 < h \leq 1$. We characterize the domains of stability for these time scales and we also give sufficient conditions for stability on a class of mixed times scale. We prove analogous results for the Samuelson model in an open economy. Our methods are based on a spectral characterization of exponential stability due to Potzsche, Siegmund and Wirth and a multiplier accelerator model on time scales due to Bohner, Gelles and Heim.

Keywords: Samuelson model; Time scales; Exponential stability.
2010 MSC: 91B64; 34N05; 34K20.

1. Introduction

In 1939, Samuelson combined the multiplier model with the acceleration principle [17]. The acceleration principle is a theory which states that small changes in the demand for consume goods can generate large changes in the demand for investment goods needed for their production. If I is the induced investment and C is the consumption, then this principle relates these variables in the following way

$$I_t = \beta \triangle C_t = \beta(C_t - C_{t-1}), \tag{1.1}$$

where in this equation, β denotes the *accelerator coefficient* with $\beta > 0$.
On the other hand, the multiplier model states the following relation

$$C_t = bY_{t-1}, \tag{1.2}$$

where Y is the national income and b is called *multiplier*, which corresponds to the propensity to spend money with $0 < b \leq 1$. Also, the following equilibrium condition holds

$$Y_t = C_t + I_t + G_t, \tag{1.3}$$

where G is the government expenditure. Using (1.1) and (1.2) Samuelson derived from (1.3) the following linear second order difference equation

$$Y_t - b(1+\beta)Y_{t+1} + b\beta Y_{t-2} = G_t. \tag{1.4}$$

[1]C. Lizama is partially supported by CONICYT under FONDECYT grant number 1140258 and Anillo ACT 1416.