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An improved spectral meshless radial point interpolation for a class of time-dependent fractional integral equations: 2D fractional evolution equation

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Abstract

In this paper, a kind of spectral meshless radial point interpolation (SMRPI) technique is applied to the fractional evolution equation in two-dimensional for arbitrary fractional order. The applied approach is based on erudite combination of meshless methods and spectral collocation techniques. The point interpolation method with the help of radial basis functions is used to construct shape functions which play as basis functions in the frame of SMRPI. In the current work, the thin plate splines (TPS) are used as the basis functions, and also the discretization technique employed is patterned after an idea of Ch. Lubich. It is proved the scheme is unconditional stable with respect to the time variable in L^2-norm and convergent by the order of convergence $\mathcal{O}(\delta t^{1+\alpha})$, $0 < \alpha < 1$. The results of numerical experiments are compared with analytical solution to confirm the accuracy and efficiency of the presented scheme. Three numerical examples, show that the SMRPI has reliable accuracy in irregular domains.

Keywords: Fractional evolution equation; Spectral meshless radial point interpolation (SMRPI) method; Radial basis function; Finite difference scheme.

1. Introduction

In recent years there has been a growing interest in the field of fractional calculus [1, 2]. Fractional differential equations have attracted increasing attention because they have applications in various fields of science and engineering [3]. Many phenomena in fluid mechanics, viscoelasticity, chemistry, physics, finance and other sciences can be described very successfully by models using mathematical tools from fractional calculus, i.e., the theory of derivatives and integrals of fractional order. Some of the most applications are given in the book of Oldham and Spanier [1], the book of Podlubny [2] and the papers of Metzler and Klafter [4], Bagley and Trovik [5]. Also a comprehensive overview of the development history of fractional calculus has been given in [6].

The present paper considers the two-dimensional fractional evolution equation [7, 8, 9, 10, 11]:

$$\frac{\partial u(\mathbf{x},t)}{\partial t} - I^{(\alpha)}\Delta u(\mathbf{x},t) = f(\mathbf{x},t), \qquad \mathbf{x}\in\Omega\subset\mathbb{R}^2, \qquad t\in(0,T], \tag{1}$$

with initial condition

$$u(\mathbf{x},0) = u_0(\mathbf{x}), \qquad \mathbf{x}\in\overline{\Omega} = \Omega\cup\partial\Omega, \tag{2}$$

and boundary condition

$$u(\mathbf{x},t) = h(\mathbf{x},t), \qquad \mathbf{x}\in\partial\Omega, t>0, \tag{3}$$

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