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Numerical Approximation of the Spatially Homogeneous Fokker-Planck-Landau Equation

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Abstract

A numerical method is developed for the spatially homogeneous Fokker-Planck-Landau (FPL) equation for the case of Coulomb collisions. The equation is put into a form so that finite difference methods for parabolic type PDE's can be applied. Through a transformation of variables the velocity space computation is reduced to a bounded domain, and finite differencing is done with a well defined, exact, boundary condition corresponding to $|v|$ at infinity. For the discretization in time implicit differencing is used. An analysis is carried out on the discrete approximation as applied to a linear FPL equation, and for the linear equation it is proved that the numerical method is unconditionally stable and convergent. Computational work is then done that demonstrates the unconditional stability and accuracy of the numerical method for the nonlinear FPL equation.

Key words. collisional plasma, Fokker-Planck-Landau equation, numerical method

AMS subject classifications. 65M06,35Q84,82D10

I Introduction

In the present paper a finite difference approximation is developed for the spatially homogeneous Fokker-Planck-Landau equation. The more relevent set of equations to physics is the spatially nonhomogeneous Vlasov-Fokker-Planck–Landau system with internally consistent electric or electromagnetic fields; however, before getting to these problems it can be useful to study the numerical approximation of the Landau collision operator by itself. Therefore, we start with the spatially homogeneous case. The initial value problem under consideration is

$$\frac{\partial f}{\partial t} = q\Phi(f,f), \quad f(v,0) = f_0(v), \tag{1.1}$$

where

$$\Phi(f,f) = \nabla_v \cdot \int_{R^3} \phi(v-v')[\nabla_v f(v) f(v') - \nabla_{v'} f(v') f(v)]\, dv', \tag{1.2}$$