Accepted Manuscript

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PII: S0377-0427(17)30170-X
DOI: http://dx.doi.org/10.1016/j.cam.2017.04.015
Reference: CAM 11094

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 3 August 2015
Revised date: 9 March 2017

Strong convergence of implicit numerical methods for nonlinear stochastic functional differential equations

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Abstract The main aim of this work is to prove that the backward Euler-Maruyama approximate solutions converge strongly to the true solutions for stochastic functional differential equations with superlinear growth coefficients. The paper also gives the boundedness and mean-square exponential stability of the exact solutions, and shows that the backward Euler-Maruyama method can preserve the boundedness of mean-square moments. Finally, a highly nonlinear example is provided to illustrate the main results.

Key words Stochastic functional differential equation, Polynomial growth condition, Backward Euler-Maruyama method, Strong convergence, Boundedness

2000 MR Subject Classification 65C30

1 Introduction

Stochastic functional differential equations (SFDEs) have received increasing attention, due to their wide applications in science and engineering fields, and their asymptotic properties have been examined by much literature (see [1-8]). Since explicit solutions can rarely be obtained, especially for highly nonlinear SFDEs, numerical solutions have obtained a great deal of attention in recent years (see [9-12]). The work [13] is a noticeable contribution to the strong convergence of numerical approximation for SFDE, in which Mao firstly studied the Euler-Maruyama (EM) approximation converges with strong order of a half for SFDE under the local Lipschitz condition and the linear growth condition. Wu and Mao [14] generalized this result to neutral SFDE under similar conditions.

The linear growth condition is so restrictive that it cannot cover many interesting models. Many authors studied the convergence of numerical solutions for highly nonlinear stochastic differential systems ([15-18]). For example, Mao [16] and Milošević [17] developed convergence in probability of the EM numerical solutions to stochastic differential equations(SDEs) with constant delay and neutral SDEs with variable delay under the Khasminskii-type conditions, respectively. Zhou et al. [18] established convergence in probability of the EM approximation for highly non-

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