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On a New Family of Radial Basis Functions: Mathematical Analysis and Applications to Option Pricing

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Abstract

In this paper, we introduce a new family of infinitely smooth and "nearly" locally supported radial basis functions (RBFs), derived from the general solution of a heat equation arising from the American option pricing problem. These basis functions are expressed in terms of "the repeated integrals of the complementary error function" and provide highly efficient tools to solve the free boundary partial differential equation resulting from the related option pricing model. We introduce an integral operator with a function-dependent lower limit which is employed as a basic tool to prove the radial positive definiteness of the proposed basis functions and could be of independent interest in the RBF theory. We then show that using the introduced functions as expansion bases in the context of an RBF-based meshless collocation scheme, we could exactly impose the transparent boundary condition accompanying the heat equation. We prove that the condition numbers of the resulting collocation matrices are orders of magnitude less than those arising from other popular RBF families used in current literature. Some other properties of these bases such as their Fourier transforms as well as some useful representations in terms of positive Borel measures will also be discussed.

Keywords: Radial basis functions; Repeated integrals of complementary error function; American option; Free boundary value problems; Artificial boundary conditions; Brenstein functions; Completely monotone functions.

2010 MSC: 65M99, 91G20, 91G80

1. Introduction

Since the path breaking and Nobel-prize winning contributions of Black and Scholes [1] and Merton [2] in 1973 to the theory of derivative security pricing and the subsequent widespread attention to options markets from practitioners and academia (see e.g. [3]), the pricing and hedging of American-style options have been an active and rapidly growing field of study within the computational finance community. These financial instruments provide the holder with the right but

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