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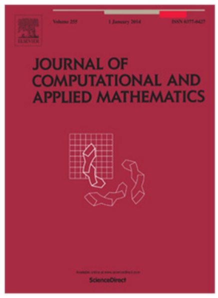

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SOME PROPERTIES OF CENTRAL DELANNOY NUMBERS

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ABSTRACT. In the paper, by investigating the generating function of central Delannoy numbers, the authors establish several explicit expressions, including determinantal expressions, for central Delannoy numbers, present three identities involving the Cauchy products of central Delannoy numbers, discover an integral representation for central Delannoy numbers, find (absolute) monotonicity, convexity, and logarithmic convexity for the sequence of central Delannoy numbers, and construct several product and determinantal inequalities for central Delannoy numbers.

1. Main results

The Delannoy numbers $D(a,b)$ are the number of lattice paths from $(0,0)$ to (a,b) in which only east $(1,0)$, north $(0,1)$, and northeast $(1,1)$ steps are allowed. They have the generating function

$$\frac{1}{1-x-y-xy}=\sum_{p,q=0}^{\infty}D(p,q)x^py^q.$$

Taking $n=a=b$ gives central Delannoy numbers $D(n)\equiv D(n,n)$, which are the number of "king walks" from the $(0,0)$ corner of an $n\times n$ square to the upper right corner (n,n). Central Delannoy numbers $D(n)$ have the generating function

$$G(x)=\frac{1}{\sqrt{1-6x+x^2}}=\sum_{n=0}^{\infty}D(n)x^k=1+3x+13x^2+63x^3+\cdots. \tag{1.1}$$

For more information on the Delannoy numbers $D(a,b)$ and central Delannoy numbers $D(n)$, please refer to the papers [8, 20] and closely-related reference therein.

It is well known [19, 22] that

(1) a sequence $a_k, k\ge 0$ is said to be increasing if and only if $a_k\le a_{k+1}$ for all $k\ge 0$;
(2) a sequence $a_k, k\ge 0$ is said to be convex if and only if $a_k+a_{k+2}\ge 2a_{k+1}$ for all $k\ge 0$; see [19, p. 42, Exercise 1] and [22, Definition 1.11];
(3) a positive sequence $a_k, k\ge 0$ is said to be logarithmically convex if and only if $a_ka_{k+2}\ge a_{k+1}^2$ for all $k\ge 0$; see [19, p. 70, Exercise 5].
(4) if the sequence $a_k, k\ge 0$ is increasing (or convex, logarithmically convex, respectively), then the function whose graph is the polygonal line corner

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Key words and phrases. central Delannoy number; explicit expression; identity; Cauchy product; integral representation; monotonicity; absolute monotonicity; convexity; logarithmic convexity; determinantal inequality; product inequality; majorization; Cauchy integral formula.

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