# Accepted Manuscript


Heterogeneous multiscale method for optimal control problem governed by parabolic equation with highly oscillatory coefficients

Liang Ge, Yanzhen Chang, Danping Yang

PII: S0377-0427(17)30329-1
DOI: http://dx.doi.org/10.1016/j.cam.2017.06.029
Reference: CAM 11207

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 7 September 2016
Revised date : 14 April 2017

Please cite this article as: L. Ge, Y. Chang, D. Yang, Heterogeneous multiscale method for optimal control problem governed by parabolic equation with highly oscillatory coefficients, *Journal of Computational and Applied Mathematics* (2017), http://dx.doi.org/10.1016/j.cam.2017.06.029

# Heterogeneous Multiscale Method for Optimal Control Problem Governed by Parabolic Equation with Highly Oscillatory Coefficients☆

Liang Ge[a], Yanzhen Chang[b,*], Danping Yang[c]

[a]*Shandong Provincial Key Laboratory of Computer Networks,Shandong Computer Science Center (National Supercomputer Center in Jinan), Jinan 250014, China*

[b]*Department of Mathematics, Beijing University of Chemical Technology, Beijing, 100029, China*

[c]*Department of Mathematics, Shanghai Key Laboratory of Pure Mathematics and Mathematical Practice, East China Normal University, Shanghai, 200062, China*

**Abstract**

In this paper, we investigate the heterogeneous multiscale method (HMM) for the optimal control problem governed by the parabolic equation with highly oscillatory coefficients. The state variable and adjoint state variable are approximated by the multiscale discretization scheme that relies on coupled macro and micro finite elements, while the control variable is discretized by the piecewise constants. By applying the well-known Lions' Lemma to the discretized optimal control problem, we obtain the necessary and sufficient optimality conditions. A priori error estimates are derived for the state, co-state and the control with uniform bounded constants. Finally, numerical results are presented to illustrate our theoretical findings.

*Keywords:* optimal control problem, heterogeneous multiscale finite element, a priori error estimate

*2010 MSC:* 49J20, 65N30

## 1. Introduction

In modern scientific and engineering computation, optimal control problems have been so frequently met in all kinds of practical problems. The research on developing fast numerical algorithms for optimal control problems is continuously and rapidly expanding, which is simply impossible to give even a very brief review here. Moreover, because the practical procedures are very complicated, many problems show the multiscale character, such as composite materials with thermal/electrical conductivity, flow through the heterogeneous porous media, and time scale of the chemical reactions, etc. Therefore, optimal control problems with multiscale character become an especially important research branch.

☆The work is supported by the Natural Science Foundation of Shandong Province, China(Grant No. ZR2011FQ030, ZR2013FQ001, ZR2013FM025), Natural Science Foundation of China(Grant No. 11501326), and the Shandong Academy of Sciences Youth Fund Project(Grant No. 2013QN007), BUCT Fund for Disciplines Construction and Development(Grant NO. XK1523).

*Corresponding author

*Email address:* gel@sdas.org(Liang Ge), changyz@mail.buct.edu.cn(Yanzhen Chang), dpyang@math.ecnu.edu.cn(Danping Yang) (Danping Yang)