# Accepted Manuscript


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PII: S0377-0427(17)30352-7
DOI: http://dx.doi.org/10.1016/j.cam.2017.07.011
Reference: CAM 11222

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 13 April 2017
Revised date : 27 June 2017

Please cite this article as: I.S. Stamatiou, A boundary preserving numerical scheme for the Wright-Fisher model, *Journal of Computational and Applied Mathematics* (2017), http://dx.doi.org/10.1016/j.cam.2017.07.011

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# A BOUNDARY PRESERVING NUMERICAL SCHEME FOR THE WRIGHT-FISHER MODEL

I. S. STAMATIOU

Abstract. We are interested in the numerical approximation of non-linear stochastic differential equations (SDEs) with solution in a certain domain. Our goal is to construct explicit numerical schemes that preserve that structure. We generalize the semi-discrete method *Halidias N. and Stamatiou I.S. (2016), On the numerical solution of some non-linear stochastic differential equations using the Semi-Discrete method, Computational Methods in Applied Mathematics,16(1)* and propose a numerical scheme, for which we prove a strong convergence result, to a class of SDEs that appears in population dynamics and ion channel dynamics within cardiac and neuronal cells. We furthermore extend our scheme to a multidimensional case.

## 1. Introduction

Let $T > 0$ and $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\leq t\leq T}, \mathbb{P})$ be a complete probability space and let $W_{t,\omega} : [0,T]\times\Omega \to \mathbb{R}$ be a one-dimensional Wiener process adapted to the filtration $\{\mathcal{F}_t\}_{0\leq t\leq T}$. We are interested in the numerical approximation of the following scalar stochastic differential equation (SDE),

$$x_t = x_0 + \int_0^t (k_1 - k_2 x_s)ds + k_3\int_0^t \sqrt{x_s(1-x_s)}dW_s, \tag{1.1}$$

where $k_i > 0, i = 1,2,3$. A boundary classification result, see Appendix A, implies that $0 < x_t < 1$ a.s. when $x_0 \in (0,1)$ and $0 < k_1 < k_2$. We therefore aim for a numerical scheme which apart from strongly converging to the true solution of (1.1), produces values in the same domain, i.e. in $(0,1)$. In other words, we are interested in numerical schemes that have an *eternal life time.*

**Definition 1.1** *[Eternal Life time of numerical solution] Let $D \subseteq \mathbb{R}^d$ and consider a process $(X_t)$ well defined on the domain $\overline{D}$, with initial condition $X_0 \in \overline{D}$ and such that*

$$\mathbb{P}(\{\omega \in \Omega : X(t,\omega) \in \overline{D}\}) = 1,$$

*for all $t > 0$. A numerical solution $(Y_{t_n})_{n\in\mathbb{N}}$ has an* eternal life time *if*

$$\mathbb{P}(Y_{n+1} \in \overline{D} \,\big|\, Y_n \in \overline{D}) = 1.$$

□

In [1] the main interest is in the domain $D = \mathbb{R}_+$. Moreover, it is clear that the Euler-Maruyama scheme has always a finite life time.

*Date*: June 27, 2017.

*Key words and phrases.* Explicit Numerical Scheme; Semi-Discrete Method; non-linear SDEs; Stochastic Differential Equations; Strong Approximation Error; Boundary Preserving Numerical Algorithm; Wright-Fisher Model.

**AMS subject classification 2010:** 60H10, 60H35, 65C20, 65C30, 65J15, 65L20, 92D99.

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