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Option pricing using a computational method based on reproducing kernel

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ABSTRACT

One of the most important subject in financial mathematics is the option pricing. The most famous result in this area is Black–Scholes formula for pricing European options. This paper is concerned with a method for solving a generalized Black–Scholes equation in a reproducing kernel Hilbert space. Subsequently, the convergence of the proposed method is studied under some hypotheses which provide the theoretical basis of the proposed method. Furthermore, the error estimates for obtained approximation in reproducing kernel Hilbert space are presented. Finally, a numerical example is considered to illustrate the computation efficiency and accuracy of the proposed method.

1. Introduction

In option pricing theory, the Black–Scholes equation is one of the most effective models for option pricing [1]. The main conceptual idea of Black and Scholes lies in the construction of a riskless portfolio taking positions in bonds (cash), option and the underlying stock. Many generalizations of this basis model have been derived in the recent decades [1]. Let us consider the generalized Black–Scholes equation

$$\frac{\partial \mathcal{C}(S,t)}{\partial t} + (r-d)S\frac{\partial \mathcal{C}(S,t)}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \mathcal{C}(S,t)}{\partial S^2} - r\mathcal{C}(S,t) = 0, \qquad (S,t) \in \Re^+ \times (0,T), \tag{1}$$

equipped with the terminal condition

$$\mathcal{C}(S,T) = (S-E)^+, \quad S \in \Re^+, \tag{2}$$

where $\mathcal{C}(S,t)$ is the value of the European call option, t is the current data, S is the asset price, T is the maturity data, d denotes the dividend of the dividend-paying asset, $r > 0$ is the risk-free interest rate, $\sigma > 0$ represents the volatility function of the underlying asset, E denotes the strike price for the option and the function $(S-E)^+$ gives the larger value between $S-E$ and 0.

If $S = 0$ at maturity data the payoff is zero even if there is a long time to maturity data, thus the call option is worthless. Hence on $S = 0$ we have

$$\mathcal{C}(0,t) = 0, \quad t \in [0,T]. \tag{3}$$

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It can be shown that the behaviour of $\mathcal{C}$ as $S \to \infty$ is

$$\mathcal{C}(S, t) \sim Se^{-d(T-t)} - Ee^{-r(T-t)}, \quad t \in [0, T]. \tag{4}$$

During the last decades, various kinds of computational techniques have been developed for the valuation of European and American options. Cox et al. [2] proposed the lattice method for option pricing, the authors of [3], used the control variate technique for option pricing, which is equivalent to an explicit time-stepping scheme, Cen and Le [4] presented a numerical method based on a central difference spatial discretization for a generalized Black–Scholes equation, the authors [5] applied a robust finite difference scheme on a piecewise uniform mesh for pricing American put options, Wang [6] and Angermann and Wang [7] proposed a fitted finite volume method for the discretization of the Black–Scholes equation governing option pricing, Valkov [8] proposed a fitted finite volume method for solving a generalized Black–Scholes equation transformed on finite interval, also Valkov in [9] presented a convergence analysis on a positivity-preserving fitted finite volume element method for a generalized Black–Scholes equation transformed on finite interval, Kadalbajoo et al. [10] implemented the uniform cubic B-spline collocation method to find the numerical solution of the generalized Black–Scholes partial differential equation.

A survey on the existence and uniqueness of solution of the stochastic equations can be found in [11,12]. Ghany et al. used Banach contraction principle method, fixed point theory, Fourier analysis and some basic inequalities to get the desire results for the existence and uniqueness of the stochastic Kadomtsev–Petviashvili equation (KP) [12] and stochastic Zakharov–Kuznetsov equation [11]. For more details on this topic one can refer to [13]. The theory of reproducing kernels has been presented by S. Zaremba [14] in his work on boundary value problems for harmonic and biharmonic functions. This theory has been successfully applied to fractal interpolation [15], solving ordinary differential equations [16–19] and partial differential equations [20,21]. The book [22] provides an excellent overview of the existing reproducing kernel methods for solving various model problems such as integral and integro-differential equations.

The main objective of the present paper is to obtain an approximate solution of the generalized Black–Scholes equation in a reproducing kernel space. The advantages of the approach lie in the following facts.

The approximate solution converges uniformly to the exact solution and the method is mesh-free, easily implemented and capable in treating various boundary conditions.

The structure of this paper is organized as follows:

The transformed problem is described in the next section, where we discuss our basic assumptions and simplify the model problem. In Section 3, we present some standard definitions and results used throughout this paper. In Section 4, we present our main results concerning our method. We give an analysis of the proposed method in this section. In Section 5, we report our numerical finding to demonstrate the accuracy and applicability of the proposed method by considering an example. Finally, we end the paper with few concluding remarks in Section 5.

2. Problem formulation

In this paper, the value of European call option is taken as a solution of Eq. (1) defined on a truncated domain $(0, S_{\max}) \times (0, T)$ with the following terminal and boundary conditions

$$\mathcal{C}(S, T) = (S - E)^+, \quad S \in [0, S_{\max}], \tag{5}$$

$$\mathcal{C}(0, t) = 0, \quad t \in [0, T], \tag{6}$$

$$\mathcal{C}(S_{\max}, t) = S_{\max}e^{-d(T-t)} - Ee^{-r(T-t)}, \quad t \in [0, T], \tag{7}$$

where $S_{\max}$ is a suitably chosen positive number.

2.1. Non-degenerate and forward in time problem

We remark that the above problem is backward in time. We can apply a time reversal and transform it to a forward time problem by the change of variable $\tau = T - t$.

In the sequel, it is convenient to transform the variable S to ζ by the change of independent variable

$$\zeta = \ln(S), \tag{8}$$

and the variable $\mathcal{C}(S, t)$ to $C(\zeta, \tau)$ by the transformation

$$\mathcal{C}(S, t) = C(\zeta, \tau). \tag{9}$$