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A Tikhonov regularized penalty function approach for solving polylinear programming problems

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Abstract

This paper suggests a new regularized penalty method for poly-linear functions. Until our knowledge it is the first time that a regularization approach solution for poly-linear programming is reported in the literature. We propose a penalty function depending on two parameters μ and δ for ensuring the strong convexity and the existence of a unique solution involving equality and inequality constraints. We prove that if the penalty parameter μ tend to zero then the solution of the original problem converge to a unique solution with the minimal weighted norm. We introduce a recurrent procedure based on the projection-gradient method for finding the extremal points and we also prove the convergence of the method. We develop an example for game theory and additional example for portfolio optimization employing the proposed regularization method for Markov chains involving the definition of a poly-linear function.

Keywords: Tikhonov, regularization, poly-linear programming, ill-posed problem, Markov chains.
2000 MSC: 65F22, 60J10

1. Introduction

1.1. Brief review

Tikhonov's regularization [33, 34] is one of the most popular approaches to solve discrete ill-posed of the minimization problem. The method looks for establishing an approximation of x by replacing the minimization of an ill-posed problem

$$\min_{x \in X_{adm}} f(x)$$

by a penalized problem of the form

$$f_\delta(x) = f(x) + \frac{\delta}{2} \|x\|^2 \quad (1)$$

where $\|\cdot\|$ denotes the Euclidean vector norm and the scalar $\delta > 0$ is known as the regularization parameter. The term $\frac{\delta}{2} \|x\|^2$ penalizes large values of x, and result in a sensible solution in cases when minimizing the first term only does not. Solving Eq. (1) requires both the determination of a value of $\delta > 0$ and the computation of $x = x_\delta$ of the minimization problem.

Regularization is used in several contexts. For instance, the Eq. (1) can be expressed as a vector optimization problem with two objectives

$$\min_{x \in X_{adm}} \left(f(x), \frac{\delta}{2} \|x\|^2\right) \quad (2)$$

where any minimum norm point in (2) is Pareto optimal and it is unique [9]. In game theory, regularization plays a fundamental role in order to ensure the convergence to one of the Nash equilibria [35] (see Section 5: Regularization example).

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