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Marching schemes for inverse scattering problems in waveguides with curved boundaries[☆]

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HIGHLIGHTS

- A marching method is developed for Cauchy problems of Helmholtz equation.
- The method deals easily backscattering in waveguides with curved boundaries.
- We present a proof for the principles of a marching algorithm.
- The method is extensively verified in waveguides with strong reflections.

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ABSTRACT

A marching scheme is developed for inverse scattering problems of the Helmholtz equation in waveguides with curved boundaries. We implement a local orthogonal transform to transform the irregular waveguide in physical plane into a regular rectangle in computing plane. Then the modified Helmholtz system in computational domain is piecewise solved through a second order numerical marching scheme, and we propose a spectral projector based on the truncated local propagating eigenfunction expansion to regularize the marching scheme. In the end, the marching scheme is verified by extensive numerical experiments, and it is shown that the scheme is efficient, stable and accurate in rapidly varying waveguides with curved boundaries, even when the number of propagating modes in the main propagation direction is variable.

1. Introduction

Large-scale wave propagation problems widely exist in many scientific areas, e.g. acoustics, electro-magnetism, seismic migration and other applications, where we often need to solve the Helmholtz equation in a very large scale range-dependent waveguide with curved boundaries or interfaces [1–4]. For these large-scale problems with curved boundaries, direct methods are very expensive for they result in very large indefinite linear systems. In contrast to this, marching methods are usually more acceptable in the sense of efficiency and storage space.

To marching compute these problems, we need to flatten the waveguides with some mathematical treatments in the first. The 'staircase' approximation has once been popularly adopted. But it often leads to marching computing in a very

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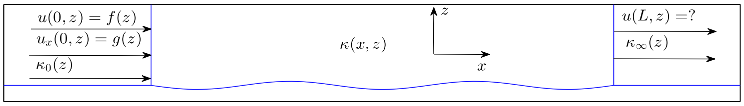

Fig. 1. The Cauchy problem sketch for a waveguide with curved bottom.

small range step size, and the marching computing is caught in a huge amount of computation trap. Alternatively, the local orthogonal transform [5,6] is often more feasible than the staircase approximation. For waveguides with internal interfaces, Zhu and Li developed an analytical local orthogonal coordinate transform and derived a modified Helmholtz equation [7,8]. Then, a numerical local orthogonal transform method (NLOTM) was also proposed to remove the divisible condition required by the analytical local orthogonal transform [9].

Then, some marching schemes can be constructed on the modified Helmholtz equation in computational plane after implementing a local orthogonal transform,.

Generally, marching methods can be categorized into two categories: the first-order methods and the second-order methods. Different from first-order methods [10–13], where first-order approximations are implemented to approximate the Helmholtz equation, second-order methods stick rather with the Helmholtz equation and can deal with backscattering. The operator marching method (OMM) [5,14,15] is an efficient second-order method in slowly varying waveguides. However, as shown in the later part of the work, it is not suitable for solving wave propagation in complex waveguides with variable number of propagating modes.

The marching methods in [16–18] are also second-order methods. But unlike the OMM, they are only restricted to inverse scattering problems or Cauchy problems [15,19,20] in conjunction with inverse problems in waveguides. Natterer and Wübbeling utilize the fast Fourier transform to filter the marching solution with a carefully determined bandwidth [17]. While in [18], an algorithm for downward extrapolation is presented to suppress only the evanescent waves with a spectral projector. For convenience, we call the marching method with the fast Fourier transform "the Fourier marching method" and the marching method with spectral projector "the spectral projector marching method". Both the marching methods are not restricted to small propagation angles, and can deal easily with backscattering.

Inverse scattering problems or Cauchy problems of the Helmholtz equation are highly ill posed, which leads to great illnesses arising in every marching step of marching methods for Cauchy problems. Therefore, some mathematical treatments have to be implemented for obtaining a physically meaningful solution. As shown by Natterer and Wübbeling [17], the stabilization of the Fourier marching method can be achieved simply by suppressing the evanescent waves, and the corresponding low-pass filtered solution will be very close to the true solution, provided that the parameter to cut off the frequency is correctly chosen. However, they also point out the error estimate needs to be formulated much tighter under appropriate conditions, since the error bound is exponentially grown and only a loss in bandwidth can make the exponent not too big. In fact, a similar method has been used to compute Cauchy problems of the Maxwell equations by Vöegeler in [21] in 2003, where the stability is maintained through restricting the solutions to spatial frequencies slightly lower than a cut off frequency.

Sandberg and Beylkin [18] more clearly demonstrate the causes for the instability of Cauchy problems. They attribute the instability of an elliptic equation to the unwanted amplification of evanescent waves, and their marching method projects the marching solution into a subspace composed only by propagating eigenfunctions. For practical use of the method, a simple matrix polynomial recursion is then used to accelerate the computing of the spectral projector, which avoids the expensive construction of eigensystems [22]. In addition, for three-dimensional problems, [23] also points out that a reasonable speed for computing spectral projectors can be obtained through using PLR (Partitioned Low Rank) representation of matrices.

However, it should be noticed that these works only deal with Cauchy problems in regular waveguides. In practical applications, more waveguides are irregular. To this end, this work mainly concentrate on general Cauchy problems in large scale complex waveguides with curved boundaries or interfaces. It is our purpose to develop an efficient and stable marching method for Cauchy problems in such irregular waveguides. The strategy is planed as follows: we first implement a local orthogonal transform to transform the irregular waveguides into a regular domain; then we build our marching scheme on the modified Helmholtz system in the computational plane. We also attempt to provide a general theoretical proof for basic principles applied to marching methods associated with Cauchy problems of the Helmholtz equation in irregular waveguides.

The paper is arranged as follows. The basic mathematical formulations are described in Section 2. In Section 3, we derive the stability and accuracy condition for Cauchy problems of the modified Helmholtz equation, and propose the marching scheme for waveguides with curved boundaries. Section 4 presents some numerical results in various irregular waveguides. We conclude our work with some discussions in Section 5.

2. Mathematical formulation

In this section, we first present the basic mathematical formulation of the Cauchy problem in waveguides with curved boundaries (see Fig. 1). Then a local orthogonal transform is implemented to flatten the curved boundaries, and the modified Helmholtz equation is obtained correspondingly. In the end, we introduce the transverse operator of the modified Helmholtz equation and its characteristic problem on the transformed computational domain.