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PII: S0377-0427(17)30294-7
DOI: http://dx.doi.org/10.1016/j.cam.2017.05.041
Reference: CAM 11172

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 28 February 2017
Revised date : 27 April 2017

Please cite this article as: T. Hasegawa, H. Sugiura, Uniform approximation to Cauchy principal value integrals with logarithmic singularity, *Journal of Computational and Applied Mathematics* (2017), http://dx.doi.org/10.1016/j.cam.2017.05.041

Uniform approximation to Cauchy principal value integrals with logarithmic singularity

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Abstract

An approximation of Clenshaw-Curtis type is given for Cauchy principal value integrals of logarithmically singular functions $I(f;c) = \fint_{-1}^{1} f(x)\ (\log|x-c|)/(x-c)\,dx$ $(c \in (-1,1))$ with a given function f. Using a polynomial p_N of degree N interpolating f at the Chebyshev nodes we obtain an approximation $I(p_N;c) \cong I(f;c)$. We expand p_N in terms of Chebyshev polynomials with $O(N \log N)$ computations by using the fast Fourier transform. Our method is efficient for smooth functions f, for which p_N converges to f fast as N grows, and so simple to implement. This is achieved by exploiting three-term inhomogeneous recurrence relations in three stages to evaluate $I(p_N;c)$. For $f(z)$ analytic on the interval $[-1,1]$ in the complex plane z, the error of the approximation $I(p_N;c)$ is shown to be bounded uniformly. Using numerical examples we demonstrate the performance of the present method.

Keywords: Cauchy principal value integral, logarithmic singularity, quadrature rule, Chebyshev interpolation, uniform approximation, three-term recurrence relations

2010 MSC: 65D30, 65D32

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