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On a new class of special functions generated by integral-difference operators

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Abstract. We introduce and investigate a new class of special functions $\Xi_N^{[\mathbf{k}]}(x)$, $x \in [0,1)$. Originally these functions naturally appeared upon spectral analysis of integral-difference operators. We discuss issues of generating functions; decompositions in series; links to Legendre polynomials, complete elliptic integrals and Gauss hypergeometric functions; functional relations for functions $\Xi_N^{[\mathbf{k}]}$, mostly in case $N = 1$.

Keywords: special functions; functional relations; decomposition in series; integral-difference operators

1. Introduction and background

Originally, a specific class of integral-difference operators K_φ had appeared as collision operators in some non-equilibrium statistical physics models [AT], [P] , [PO]. Their rigorous mathematical study in dimension 1 started in late 1990-s [M1-M5], [MY] and has been continued later [Lo], [M6]. Spectral analysis of these operators has demonstrated encouraging links with various fields of mathematics [M5], [M6]: functional analysis, asymptotic analysis, deviation functionals, orthogonal polynomials, Jacobi matrices etc. Also, another important physical application (matter relaxation processes in a field) has been traced [M6]. There operators K_φ determine the dynamics of matter relaxation processes in external attractive field $\varphi(x)$. Similar spectral problems occur also in the models of the electronic transport through resistive multichannel quantum wires [GL], [Kn], [Ku1 - Ku3], [Le1], [Le2].

Recent generalisation to higher dimensions [M6] had led to the appearence of a new class of special functions $\Xi_N^{[\mathbf{k}]}$, which are the subject of the present paper.

The above mentioned operators act in Hilbert space $\mathcal{H} = L_2(\Omega)$, where $\Omega \subset \mathbf{R}^N$ is a compact N-dmensional domain with a smooth boundary, as

$$K_\varphi : u(\mathbf{x}) \mapsto \int_\Omega \frac{u(\mathbf{x})\varphi(\mathbf{s}) - u(\mathbf{s})\varphi(\mathbf{x})}{|\mathbf{x} - \mathbf{s}|} \, d^N\mathbf{s} \, . \tag{1}$$