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An interior affine scaling cubic regularization algorithm for derivative-free optimization subject to bound constraints[☆]

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ABSTRACT

In this paper, we introduce an affine scaling cubic regularization algorithm for solving optimization problem without available derivatives subject to bound constraints employing a polynomial interpolation approach to handle the unavailable derivatives of the original objective function. We first define an affine scaling cubic model of the approximate objective function which is obtained by the polynomial interpolation approach with an affine scaling method. At each iteration a candidate search direction is determined by solving the affine scaling cubic regularization subproblem and the new iteration is strictly feasible by way of an interior backtracking technique. The global convergence and local superlinear convergence of the proposed algorithm are established under some mild conditions. Preliminary numerical results are reported to show the effectiveness of the proposed algorithm.

1. Introduction

In this paper, we analyze the solution of the following nonlinear derivative-free minimization subject to bound constraints:

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & f(x) \\ \text{s.t.} \quad & l \le x \le u, \end{aligned} \tag{1.1}$$

where $f(x):\ \mathbb{R}^n \to \mathbb{R}$ is at least continuous in the feasible region, and its first-order and second-order derivatives may be unavailable or unreliable, and the vectors $l \in (\mathbb{R} \cup \{-\infty\})^n$ and $u \in (\mathbb{R} \cup \{+\infty\})^n$ are specified lower and upper bounds. We define $\mathcal{F} \stackrel{\text{def}}{=} \{x \in \mathbb{R}^n \mid l \le x \le u\}$ as the feasible set and $\text{int}(\mathcal{F}) \stackrel{\text{def}}{=} \{x \in \mathbb{R}^n \mid l < x < u\}$ as the strictly feasible set of problem (1.1).

In the literature, many approaches for derivative-based nonlinear programming have been extended to a derivative-free context. In [1–5], a class of derivative-free trust region methods for solving unconstrained optimization has been proposed. These methods are based on the sequential minimization of quadratic (or linear) models built from evaluating the objective function at sample sets. In [6], the author worked in bound constrained optimization without derivatives of the objective function. The derivative-free trust region method was adapted to include an affine scaling strategy with the interior backing line search technique for solving nonlinear systems subject to linear equality constraints in [7]. In order to increase the tools

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available for the solution of constrained problems where derivatives of the objective function are not available, we extend the cubic regularization algorithm to solve the derivative-free optimization subject to bound constraints based on a polynomial interpolation approach.

Recently, an adaptive regularization approach with cubic (shortened to ARC) has been proposed by Cartis et al. [8,9] as a new globalization technique for unconstrained optimization. First ideas on ARC approaches can be found in [10] in the context of affine invariant Newton methods. Further results were obtained by Nesterov et al. in [11] where it has shown that the use of a local cubic overestimator for f yields an algorithm with global and fast local convergence properties. In [12], Bellavia et al. studied adaptive regularized methods for nonlinear least-squares problems. In this latter paper the model of the objective function used at each iteration is either the Euclidean residual regularized by a quadratic term or the Gauss–Newton model regularized by a cubic term. Quadratic convergence to zero-residual solutions was proved under an error-bound condition. The ARC algorithm considered by Cartis et al. [8] for unconstrained optimization problems consists in solving a sequence of approximate minimizations of a quadratic model regularized by a local cubic term. The regularization term is adaptively chosen and it is related to the local Lipschitz constant of the objective function Hessian. Its role is similar to the role that the trust-region radius plays in trust region methods for unconstrained optimization. The regularization term is decreased whenever the search direction is accepted. The method has been shown to have excellent global and local convergence properties comparing with the trust region approach when solving small scale problems. In [13], Cartis et al. modified the ARC method described in [8] to solve the problem of minimizing a nonlinear, possibly nonconvex, smooth objective function over a convex domain. Convergence to first-order critical points was shown under standard assumptions without any Lipschitz continuity requirement on the objective's Hessian. In the context of simple bound optimization, a possible way to overcome the lack of bound constraint information can be to use an affine scaling technique. In [14], a well-known affine scaling algorithm for solving (1.1) with available derivatives was proposed. Based on an equivalent scaled Karush–Kuhn– Tucker (KKT) system, the authors minimized a quadratic function subject to an ellipsoidal constraint. The affine scaling algorithm was first considered by Dikin [15] for linear programming. Many papers have been published on affine scaling algorithms for quadratic programming problems [16–18]. For general nonlinear programming problems, affine scaling algorithms normally use trust region technique to ensure convergence. To the best of our knowledge, affine scaling methods embedded in a cubic regularization framework which possesses excellent convergence results have never been proposed, besides, to enforce feasibility, this requires a suitable line search in combination with a possible step-back that is necessary to stay strictly feasible. In [13], an approximate minimization of a cubic model is carried out using a generalized Goldstein-like line search along a Cauchy arc that defined by the projection onto the feasible region of the negative gradient path. The aim is to obtain the generalized Cauchy point. The approach proposed here seems to be easier to code than the above projection method involving the (unique) orthogonal projector. What is more, at each step, the subproblem is solved only once thanks to the combination of the ARC model and line search strategies.

In this paper, under some standard assumptions in a constrained context, we introduce an affine scaling cubic regularization with interior backtracking technique algorithm for solving derivative-free optimization subject to bound constraints. Due to the unavailable derivatives of the objective function, an approximate objective function is determined by employing a polynomial interpolation approach. Motivated by the affine scaling methods and the ARC algorithms, we define an affine scaling cubic model of the approximate objective function to be minimized for generating a trial step which is modified to be strictly feasible using a suitable combination of a line search method and a necessary step-back technique as we require that every iteration must be a strictly feasible point. Moreover, we find an interesting relationship among the scaling matrix, the interpolating radius and the gradient which plays a significant role in the global and superlinear convergence proofs. The excellent convergence results are finally retained. There are two advantages of this proposed algorithm, one is that we only need to solve the subproblem once at each iteration if the interpolating radius meets the desirable condition, while the derivative-based algorithm in [14] needs to solve the subproblem repeatedly if the trial step is not accepted; and the other is that the new iteration is strictly feasible as the approximate minimizer of the affine scaling cubic subproblem is modified by a monotone interior line search and a possible step-back. Finally, numerical results show that the algorithm proposed in this paper are promising.

The outline of the paper is as follows. In Section 2 we recall basic assumptions, some preliminary results and introduce the basic definitions of the interpolation models, some basic properties of the interpolation models required to prove the general convergence result. Then we describe an affine scaling adaptive cubic regularization algorithm for derivative-free optimization subject to bound constraints. Besides, we give the global convergence results in Section 3. In Section 4 we show some local convergent results. The numerical performances of our algorithm are provided in Section 5.

As a general rule for notation, $\|\cdot\|$ is the 2-norm for a vector and the induced 2-norm for a matrix. For any vector $x \in \mathbb{R}^n$, we use the notation $[x]_i$ or x_i to denote the ith component of the vector x, $\nabla f(x)$ to denote the gradient of function f on the variable x and $\nabla^2 f(x)$ to denote the Hessian matrix of function f on the variable x. The subscript k denotes an index for a sequence.

2. Algorithm

In this section, we intend to describe an affine scaling cubic regularization with interior backtracking technique algorithm for derivative-free optimization subject to bound constraints. We first recall a simple optimality condition for the optimization problem (1.1).