

## Accepted Manuscript

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PII: S0377-0427(17)30086-9

DOI: <http://dx.doi.org/10.1016/j.cam.2017.02.021>

Reference: CAM 11023

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 24 October 2016

Revised date: 20 January 2017



Please cite this article as: M. Akbas, M. Mohebujjaman, L.G. Rebholz, M. Xiao, High order algebraic splitting for magnetohydrodynamics simulation, *Journal of Computational and Applied Mathematics* (2017), <http://dx.doi.org/10.1016/j.cam.2017.02.021>

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# High order algebraic splitting for magnetohydrodynamics simulation

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March 1, 2017

## Abstract

This paper proposes, analyzes and tests high order algebraic splitting methods for magnetohydrodynamic (MHD) flows. The main idea is to apply, at each time step, Yosida-type algebraic splitting to a block saddle point problem that arises from a particular incremental formulation of MHD. By doing so, we dramatically reduce the complexity of the nonsymmetric block Schur complement by decoupling it into two Stokes-type Schur complements, each of which is symmetric positive definite and also is the same at each time step. We prove the splitting is  $O(\Delta t^3)$  accurate, and if used together with (block-)pressure correction, is fourth order. A full analysis of the solver is given, both as a linear algebraic approximation, but also in a finite element context that uses the natural spatial norms. Numerical tests are given to illustrate the theory and show the effectiveness of the method.

## 1 Introduction

The flow of electrically conducting fluids in the presence of a magnetic field is called magnetohydrodynamics (MHD) flow. Such flows arise, for example, in astrophysics and geophysics [15, 21, 10, 9, 3, 4], liquid metal cooling of nuclear reactors [2, 13, 26], and process metallurgy [8]. However, simulation of these flows is known to be quite difficult, and one major reason for this is the difficulties that arise because of the large, nonsymmetric, ill-conditioned block saddle point linear systems that arise at each time step. It is the purpose of this paper to propose, analyze and test an accurate and efficient linear solver for these systems, by extending some recent work of the authors on saddle point linear systems for Navier-Stokes (NS) [24] to the block saddle point systems that arise in MHD. The key ideas are combining the Yosida algebraic splitting with a particular incremental formulation of the MHD system at each time step, which leads to a Schur complement (the main difficulty of the linear solve) that decouples into two Stokes-type Schur complements, each of which are symmetric positive definite, and are also the same at each time step. We will fully analyze the splitting error and show it is third order (fourth order if block pressure-correction is applied), and to our knowledge, this is the first higher order algebraic splitting method studied for the block saddle point systems in MHD.

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