



Phase-fitted Runge–Kutta pairs of orders 8(7)

Ch. Tsitouras^a, I.Th. Famelis^b, T.E. Simos^{c,d,*}^a *TEI of Sterea Hellas, Department of Automation Engineering, GR34400, Psahna Campus, Greece*^b *TEI of Athens, microSENSES Laboratory, Department of Electronic Engineering, GR11210, Egaleo, Greece*^c *Group of Modern Computational Methods, Ural Federal University, 620002, 19 Mira Street, Ekaterinburg, Russian Federation*^d *Laboratory of Computational Sciences, Department of Informatics and Telecommunications, Faculty of Economy, Management and Informatics, University of Peloponnese, Greece¹*

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ABSTRACT

A new phase fitted Runge–Kutta pair of orders 8(7) which is a modification of a well known explicit Runge–Kutta pair for the integration of periodic initial value problems is presented. Numerical experiments show the efficiency of the new pair in a wide range of oscillatory problems.

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1. Introduction

For the numerical solution of the initial value problem

$$y' = f(x, y), \quad y(x_0) = y_0 \in \mathbb{R}^m, \quad x \in [x_0, x_e] \quad (1)$$

where $f : \mathbb{R} \times \mathbb{R}^m \mapsto \mathbb{R}^m$, the Explicit Runge–Kutta (RK) pairs are widely used. Such pairs can be presented by the extended Butcher tableau [1,2]:

$$\begin{array}{c|c} c & A \\ \hline & b \\ & \hat{b} \end{array}$$

with $b^T, \hat{b}^T, c \in \mathbb{R}^s$ and $A \in \mathbb{R}^{s \times s}$ is strictly lower triangular. Such methods advance the solution from x_n to $x_{n+1} = x_n + h_n$ using the following two approximations at each step, y_{n+1}, \hat{y}_{n+1} to $y(x_{n+1})$ of orders p and $p - 1$ respectively,

$$y_{n+1} = y_n + h_n \sum_{i=1}^s b_i f_{ni}$$

* Correspondence to: 26 Menelaou Street Amfithea-Paleon Faliron, GR-175 64 Athens, Greece.

E-mail address: tsimos.conf@gmail.com (T.E. Simos).

URLs: <http://users.ntua.gr/tsitoura> (Ch. Tsitouras), <http://users.teiath.gr/ifamelis> (I.Th. Famelis).

¹ Part time.

and

$$\hat{y}_{n+1} = y_n + h_n \sum_{i=1}^s \hat{b}_i f_{ni},$$

where

$$f_{ni} = f \left(x_n + c_i h_n, y_n + h_n \sum_{j=1}^{i-1} a_{ij} f_{nj} \right)$$

for $i = 1, 2, \dots, s$. Using this embedded form we can obtain an estimate $E_{n+1} = \|y_{n+1} - \hat{y}_{n+1}\|$ of the local truncation error of the $p - 1$ order formula. Then the next step of the numerical solution can be computed using a step-size control algorithm

$$h_{n+1} = 0.9 \cdot h_n \cdot \left(\frac{\text{TOL}}{E_{n+1}} \right)^{1/p}, \quad (2)$$

where TOL being the requested tolerance. If $E_{n+1} > \text{TOL}$ the step h_n computed to advance the approximation of the solution from x_n to x_{n+1} is rejected and the same formula (2) is assumed to recompute the new smaller current step. In (2) the 0.9 is a safety factor that is used so that the error will be acceptable the next time with high probability. For more details on the implementation of these types of step size policies see [2,3].

2. Basic theory

When a RK method is applied to solve the test problem

$$y' = i\omega y, \quad \omega \in \Re, \quad i = \sqrt{-1}, \quad (3)$$

we are led to the numerical scheme

$$y_{n+1} = P(i\omega h_n) y_n,$$

$h_n = x_{n+1} - x_n$, where the function $P(iv) = P(i\omega h)$ satisfies the relation

$$P(iv) = 1 + ivb(I - ivA)^{-1}e = \sum_{j=0}^{\infty} t_j (iv)^j, \quad (4)$$

and for $j \geq 1$, $t_j = bA^{j-1}e$, $t_0 = 1$ and $e = [1, 1, \dots, 1] \in \Re^s$ [4]. The quantities t_j depend only on the coefficients of the method. For explicit methods (e.g. A strictly lower triangular), the above sum in the determination of $P(iv)$ is finite and j runs from 0 to s .

Following [5], we define the phase-lag (or dispersion) order of a RK method as the order of approximation of the argument of the exponential function by the argument of P along the imaginary axis. Equivalently, the phase-lag order of a method is q , whenever $\delta(v) = O(v^{q+1})$, for $\delta(v) = v - \arg(P(iv))$. We define also, the imaginary stability interval of a RK method $I_l = (0, v_0)$ by the relations $|P(iv)| < 1$ and $|P(i(v_0 + \theta))| > 1$, for every $v \in I_l$ and every suitably small positive θ . When a method has a non-vanishing imaginary stability interval then it is called dissipative.

Even though in the definition for a RK method the phase-lag property is based on the solution of a special problem (3), the numerical tests presented in [6,4] strongly indicated that the RK pairs of high phase-lag order exhibit a remarkable numerical performance on a much wider class of test problems. Especially for a certain class of initial value problems (as those whose solutions are described by free oscillations or free oscillations of low frequency with forced oscillations of high frequency superimposed, over long integration intervals), one should use pairs of methods of high phase-lag order with minimized leading truncation error coefficients instead of pairs of the same algebraic order which attain the minimal algebraic order and phase-lag order allowed by the number of method's stages.

3. Methods with known frequency

Gautschi [7] has been the first who tried to fit a method to a set of linearly independent trigonometric functions. Since then a lot of methods trying to do something similar have been constructed. Here we will exploit the knowledge of $v = \omega h$ in the direction of the ideas presented in the previous section.

We observe that

$$\begin{aligned} P(iv) &= Q(v) + iR(v) \\ &= (1 - t_2 v^2 + t_4 v^4 - t_6 v^6 \pm \dots) + i(v - t_3 v^3 + t_5 v^5 - t_7 v^7 \pm \dots) \end{aligned}$$

which is a finite series for explicit methods, as we have mentioned.

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