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A convergence analysis of a fourth-order method for computing all zeros of a polynomial simultaneously

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Abstract

In 2011, Petković, Rančić and Milošević [6] introduced and studied a new fourth-order iterative method for finding all zeros of a polynomial simultaneously. They obtained a semilocal convergence theorem for their method with computationally verifiable initial conditions, which is of practical importance. In this paper, we provide new local as well as semilocal convergence results for this method over an algebraically closed normed field. Our semilocal results improve and complement the result of Petković, Rančić and Milošević in several directions. The main advantage of the new semilocal results are: weaker sufficient convergence conditions, computationally verifiable a posteriori error estimates, and computationally verifiable sufficient conditions for all zeros of a polynomial to be simple. Furthermore, several numerical examples are provided to show some practical applications of our semilocal results.

Keywords: Simultaneous methods, Polynomial zeros, Local convergence, Semilocal convergence, Error estimates, Cone metric space
2000 MSC: 65H04, 12Y05

1. Introduction

Throughout this paper $(\mathbb{K}, |\cdot|)$ denotes an algebraically closed normed field and $\mathbb{K}[z]$ denotes the ring of polynomials over $\mathbb{K}$. Let the vector space $\mathbb{K}^n$ be endowed with the p-norm $\|\cdot\|_p \colon \mathbb{K}^n \to \mathbb{R}$ defined by $\|x\|_p = \left(\sum_{i=1}^n |x_i|^p\right)^{1/p}$ for some $1 \le p \le \infty$.

The function $d \colon \mathbb{K}^n \to \mathbb{R}^n$ is defined by $d(x) = (d_1(x), \dots, d_n(x))$, where

$$d_i(x) = \min_{j \neq i} |x_i - x_j| \qquad (i = 1, \dots, n),$$

and the function $\delta \colon \mathbb{K}^n \to \mathbb{R}_+$ is defined by

$$\delta(x) = \min_{i \neq j} |x_i - x_j|.$$

In the sequel, for two vectors $x \in \mathbb{K}^n$ and $y \in \mathbb{R}^n$ we denote by $\frac{x}{y}$ the vector in $\mathbb{R}^n$ defined by

$$\frac{x}{y} = \left(\frac{|x_1|}{y_1}, \cdots, \frac{|x_n|}{y_n}\right)$$

provided that y has only nonzero components. Throughout the paper, $\mathcal{D}$ denotes the set of all vectors in $\mathbb{K}^n$ with pairwise distinct components, i.e.

$$\mathcal{D} = \{x \in \mathbb{K}^n \colon \delta(x) > 0\}.$$

Let $f \in \mathbb{K}[z]$ be a polynomial of degree $n \ge 2$. A vector $\xi \in \mathbb{K}^n$ is called a *root vector* of polynomial f if $f(z) = a_0 \prod_{i=1}^n (z - \xi_i)$ for all $z \in \mathbb{K}$, where $a_0 \in \mathbb{K}$. The first iterative method for simultaneous computation of all

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