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An conservative anti-diffusion technique for the level set method[☆]

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ABSTRACT

A novel conservative level set method is introduced for the approximation of two-phase incompressible fluid flows. The method builds on recent conservative level set approaches and utilizes an entropy production to construct a balanced artificial diffusion and artificial anti-diffusion. The method is self-tuning, maximum principle preserving, suitable for unstructured meshes, and neither re-initialization of the level set function nor reconstruction of the interface is needed for long-time simulation. Computational results in one, two and three dimensions are presented for finite element and finite volume implementations of the method.

1. Introduction

The method of level sets was first introduced by Osher and Sethian [1] in the late 1980s as a technique for capturing evolving interfaces and tracking the propagation of fronts. The traditional level set method is useful for handling complex topological dynamics but suffers from a lack of mass conservation. In the present paper we present a novel conservative level set technique using a new compression strategy; the underlying idea is to utilize a corrective flux constructed from entropy principles. A first-order application of the approach yields a provable maximum principle. The first-order scheme can be extended to a high-order non-linear compression, and flux-corrected transport techniques can be applied to retrieve the maximum principle.

Conservative level set methods have been the focus of recent research. Techniques for enforcing mass conservation in level set methods differ depending on the context of the method. The use of an entropy-production to inform the assembly of a precise anti-diffusion performs similarly to hybrid particle-type methods (e.g., Enright et al. [2], Ianniello and Di Mascio [3]). The proposed construction is, however, less complex, and straight-forward to implement. In addition, the proposed method does not require solving a re-initialization or interface reconstruction subproblem (see e.g., Fedkiw et al. [4], Olsson and Kreiss [5]). Further, the non-compressive first-order variant of our method does not rely on heuristics such as limiters or explicit mass redistribution (see e.g., Olsson and Kreiss [5], Chiu and Lin [6]) to address the usual undershoots, and overshoots.

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The current work offers other improvements over many contemporary algorithms, and specific comparisons are discussed in the computational results, Sections 7–12. For instance, the proposed approach does not invoke a characteristic mesh size; this is important for general unstructured meshes as the notion of local mesh size can be problematic to define (see Section 2.4 in Guermond and Nazarov [7] for a discussion on this topic). Secondly, the current approach details how to construct a first-order, diffusion corrected, viscous tensor which is maximum principle preserving, as proven in Guermond and Popov [8], provided the transport velocity is incompressible. Third, a high-order extension of the diffusion corrected dissipation is explained; this extension ensures that the artificial dissipation vanishes outside of a neighborhood of the level set iso-surface. Finally, it is shown how this extension can be made maximum-principle preserving through the adaptation of the flux-corrected transport ideas of Boris–Book–Zalesak. A similar methodology has been proposed in Chiu and Lin [6, Eq. (18)] and, in some sense, our work can be viewed as an extension thereof. In a wider, more historical, context the proposed method can be seen as combining the level set method with the one-step re-initialization heuristics of Coupez [9], the theory of artificial compressors vis-a-vis Harten [10,11], Olsson and Kreiss [5] and the entropy production principles of Guermond et al. [12].

The paper is organized as follows. Section 2 gives a brief literature survey of historical topics related to the discussions in the sections that follow. Section 3 discusses the preliminaries of artificial compression as a viscous correction and introduces Eq. (10). This equation is the motivation of the method proposed in the paper and can be viewed as a one-stage version of the classical artificial compression method proposed by Harten [10,11]. Section 4 describes the full method and implementation in the context of finite elements. Section 5 describes the analogous implementation details in the finite volume framework. Section 6 details an extension of the methods to a maximum principle preserving technique; this is done by adapting the flux-corrected transport methodology of Boris–Book–Zalesak. Sections 7–12 present numerical illustrations of the performance of the method in one, two, and three dimensions for a variety of benchmark test problems. The computations and results discussed in Sections 7–12 demonstrate the efficacy of the technique for interface capturing. The computation of secondary terms, such as surface tension or stresses, is out of the scope the current work and is therefore not discussed.

2. Brief overview of the literature

This section gives a brief survey of the topics which constitute a historical foundation for the proposed method. Readers who are already familiar with numerical front tracking methods, the level set method, the role of re-initialization in the level set method, and the connection between re-initialization of Heaviside level set functions and anti-diffusion should advance to Section 3.

2.1. Numerical front tracking and the level set method

Numerical techniques for the evolution of interfaces and free surfaces are an active area of research. Popular Eulerian approaches for the transport of an interface include volume tracking methods and level set methods. More extensive sources for the historical development of these methods and their variants are found in, respectively, Rider and Kothe [13] and Osher and Fedkiw [14]. A drawback of the volume tracking methods is the difficulty in a-posteriori interface reconstruction while level set methods can suffer from loss of area enclosed by the interface. Hybrid methods have been proposed to address these issues such as coupling volume tracking and the level set method (CLSVOF) (Sussman and Puckett [15]), the addition of marker particles (Enright et al. [2]), and the extension to oriented marker particles with reconstruction (Ianniello and Di Mascio [3]).

The level set method typically refers to the transport of a smooth distance function whose zero isosurface is the interface to be tracked. The method has been used successfully in many fluid flow applications, see e.g., Sussman et al. [16], Osher and Sethian [1], Gibou et al. [17], Ville et al. [18], Bonito et al. [19], but it can also be used in other contexts like shape optimization as in Dapogny et al. [20], Yamada et al. [21].

If the advection velocity field is incompressible, the transport problem can be recast in conservative form which allows for the use of smoothed Heaviside level set functions, conservative discretizations, and anti-diffusion techniques for enhancing mass conservation.

Given a velocity field $\boldsymbol{u}$, the basic equation for the level set method is

$$\phi_t + \boldsymbol{u} \cdot \nabla\phi = 0. \tag{1}$$

Depending on the specifics of the problem, the above equation is often rewritten in alternative forms. For instance if $\boldsymbol{u}$ is divergence free, (1) can be expressed in the conservation form $\phi_t + \nabla\cdot(\boldsymbol{u}\phi) = 0$ which can be discretized using conservative numerical schemes. In the context of curvature driven flows Eq. (1) can be rewritten as follows:

$$\phi_t + u_N \|\nabla\phi\|_{\ell^2} = 0, \tag{2}$$

which is of Hamilton–Jacobi type, allowing access to fast numerical schemes for solving problems in this category. Both Eqs. (1) and (2) are often referred to as 'the level set equation'. The reader is referred to Fedkiw et al. [4], and the sources therein, for a large selection of references on the many applications of level set methods. We also refer to Dapogny et al. [20, Section 4] for a review of important mathematical properties of the signed distance function.