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A splitting preconditioner for a block two-by-two linear system with applications to the bidomain equations

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Abstract

We construct an alternating splitting iteration scheme for solving and preconditioning a block two-by-two linear system arising from numerical discretizations of the bidomain equations. The convergence theory of this class of splitting iteration methods is established and some useful properties of the preconditioned matrix are analyzed. The potential of this approach is illustrated by numerical experiments.

Key words: block two-by-two matrix, iterative methods, preconditioning, matrix splitting, bidomain equations

1. Introduction

In this work we construct preconditioners for discrete bidomain model. This model describes the electrical activity in the heart, see [18] for details. The general bidomain equations are on the following form

$$\begin{cases} \dfrac{\partial v}{\partial t} = \nabla \cdot (M_i \nabla v) + \nabla \cdot (M_i \nabla u_e) + f(v,w), \\ 0 = \nabla \cdot (M_i \nabla v) + \nabla \cdot ((M_i + M_e)\nabla u_e), \\ \dfrac{\partial w}{\partial t} = g(v,w). \end{cases} \tag{1.1}$$

Here, the unknowns are the transmembrane potential v and the extracellular potential u_e. The tensors M_i and M_e describe the intra and extra-cellular conductivities of the heart, and w denotes the state of the heart cells. Usually these equations are assigned with homogeneous Neumann boundary conditions, i.e.,

$$\mathbf{n}^T M_i \nabla (v + u_e) = 0 \quad \text{and} \quad \mathbf{n}^T (M_i + M_e)\nabla u_e + \mathbf{n}^T M_i \nabla v = 0, \tag{1.2}$$

where $\mathbf{n}$ is the outwards directed unit normal vector defined along the boundary of the heart. Note that u_e is only determined up to a constant in the case of Neumann conditions.

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