Harmonic analysis associated with the modified Cherednik type operator and quantitative uncertainty principles for its Hartley transform

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HARMONIC ANALYSIS ASSOCIATED WITH THE MODIFIED CHEREDNIK TYPE OPERATOR AND QUANTITATIVE UNCERTAINTY PRINCIPLES FOR ITS HARTLEY TRANSFORM

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Abstract. We consider a new differential-difference operator Λ on the real line. We study the harmonic analysis associated with this operator. Next, we prove various mathematical aspects of the quantitative uncertainty principles, including Donoho-Stark's uncertainty principle and variants of Heisenberg's inequalities for its Hartley transform associated to the operator Λ.

1. Introduction

Classical quantitative uncertainty principles is just another name for some special inequalities. These inequalities give us information about how a function and its Fourier transform relate. They are called uncertainty principles since they are similar to the classical Heisenberg uncertainty principle, which has had a big part to play in the development and understanding of quantum physics. For example: Benedicks [2], Slepian and Pollak [20], Slepian[21], and Donoho and Stark [7] paid attention to the supports of functions and gave quantitative uncertainty principles for the Fourier transforms.

The quantitative uncertainty principles have been studied by many authors for various Fourier transforms, see [3, 13, 14, 15, 26] for instance and the related references cited therein. It is worth mentioning that, for a class of such principles, the generalized Fourier transform should have some properties, like the inversion and Plancherel's formulas, the positivity of the spectral measure and the boundedness of its kernel. However, there are some generalized Fourier transforms that do not satisfy the previous properties, such as the generalized Fourier transform associated with the Cherednik operators (known as the Opdam-Cherednik transform), see [12, 18] and the generalized Fourier transform associated with the operator of Cherednik type, see [16].

In [6], Cherednik introduced a family of differential-difference operators that nowadays bear his name. These operators play a crucial role in the theory of Heckman-Opdam's hypergeometric functions, which generalizes the theory of Harish-Chandra's spherical functions on Riemannian symmetric spaces, see [12, 18]. For recent developments and important results in this direction we refer the reader to [19]. The one-dimensional Cherednik operator is given by

$$\Lambda_{k,k'}f(x) = \frac{d}{dx}f(x) + \big(k\coth(x) + k'\tanh(x)\big)\{f(x) - f(-x)\} - \rho f(-x),$$

where $k \geq k' \geq 0, k \neq 0$. It is known by now that harmonic analysis associated with $\Lambda_{k,k'}$ has a considerable technical difficulty to be overcome comparable to the classical harmonic analysis, see for instance [1, 8]. We also note that the operator $\Lambda_{k,k'}$ is a particular case of that of the Cherednik type, see [16].

The main problems that we are facing in the harmonic analysis associated with the one-dimensional Cherednik operator or that of the Cherednik operator type, are itemized in the fact that we are in front of spectral complex measures and the Plancherel's formula for the generalized Fourier transform, associated with the previous operators, does not occur. As impact

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