Accepted Manuscript

Wavelets for the Maxwell's equations: An overview

Sergio Amat, Pedro J. Blázquez, Sonia Busquier, Concepción Bermúdez

PII: S0377-0427(17)30073-0
DOI: http://dx.doi.org/10.1016/j.cam.2017.02.015
Reference: CAM 11017

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 14 December 2015
Revised date: 1 February 2017

Please cite this article as: S. Amat, P.J. Blázquez, S. Busquier, C. Bermúdez, Wavelets for the Maxwell's equations: An overview, *Journal of Computational and Applied Mathematics* (2017), http://dx.doi.org/10.1016/j.cam.2017.02.015

Wavelets for the Maxwell's Equations: An Overview☆

Sergio Amat[a], Pedro J. Blázquez[b,*], Sonia Busquier[a], Concepción Bermúdez[a]

[a]*Departamento de Matemática Aplicada y Estadística. Universidad Politécnica de Cartagena, Spain.*
[b]*Universidad Internacional de La Rioja, Av Gran Vía Rey Juan Carlos I, 41, 26002 Logroño, La Rioja, Spain*

Abstract

In recent years wavelets decompositions have been widely used in computational Maxwell's curl equations, to effectively resolve complex problems. In this paper, we review different types of wavelets that we can consider, the Cohen-Daubechies-Feauveau biorthogonal wavelets, the orthogonal Daubechies wavelets and the Deslauriers-Dubuc interpolating wavelets. We summarize the main features of these frameworks and we propose some possible future works.

Keywords: Wavelets, Multiresolution, Stability, Adaptivity, Maxwell's equations.

1. Introduction

In the last decades, application of the wavelets theory has been extensively investigated in various research fields of science and engineering. The wavelet decompositions yield very efficient algorithms, in terms of accuracy and CPU time, when applied to numerical solutions of differential equations. In particular, three methods of deriving wavelet schemes have been presented so far in the literature to solve electromagnetic problems from Maxwell's equations: Multiresolution Time-Domain (MRTD) scheme, Fast Wavelet Transform-based (FWT) algorithms and Interpolating Wavelets (IW). Although the three methods are time-domain schemes, they differs in how wavelet theory is implemented.

In the MRTD method the electric and magnetic fields are expanded in a wavelet basis and Maxwell'curl equations are discretized using the Garlekin's version of the method of moment [50]. Two conflicting requirements for the choice of the wavelet basis are high regularity properties and minimal support. The former reduces numerical dispersion and the latter reduces the algorithmic computational complexity and improves stability. In a first approach, we can find a formulation based on the Haar [32] or Battle-Lemarie [39], [46] orthonormal wavelets. The Haar family have compact support and it yields a simple algorithm but it lacks smoothness, then poor numerical dispersion properties are expected. In contrast, the Battle-Lemarie family have good regularity properties which yields highly linear numerical dispersion behavior, but they have infinite support, thus the MRTD scheme have to be truncated affecting the accuracy of the field computation. As a natural alternative, the orthogonal Daubechies wavelets [25] and the Cohen-Daubechies-Feauveau biorthogonal wavelets [23] have been considered. Both being compactly supported, the CDF biorthogonal wavelet family seems to allow a good balance between regularity and reduced support, while also being symmetric.

Other alternative, which we have named FWT, leading to fast and very efficient algorithms, was first proposed in [51]. It uses the fast wavelet transform and multiscale representation of derivative operators for compactly supported wavelets in the form of [16]. In contrast to the MRTD schemes, no integrals have to be evaluated. Orthogonal Daubechies'wavelets were used in the above mentioned work as well in [41] and [28]. Biorthogonal CDF wavelets were also used in [28, 48].

On the other hand, there exists a fundamental characteristic of some sort of wavelets: the interpolation property. The advantage of interpolating wavelets (IW) is that the coefficients of the associated expansion represent directly the physical values of the electromagnetic fields. The so-called "shifted interpolating property" [44] of Daubechies' wavelets has been used in [18] and [34]. The last authors also proposed in [17] a higher order biorthogonal scheme using Deslauries-Dubuc interpolating functions [26], [30] which are smooth, symmetric and compactly supported. If the basis do not have the interpolating property, then it is necessary considering some means of the neighboring coefficients or reconstruction procedure in order to obtain the physical variables, resulting in a more elaborate scheme and more computational cost.

☆This research was supported in part by by Programa de Apoyo a la investigación de la fundación Séneca-Agencia de Ciencia y Tecnología de la Región de Murcia 19374/PI/14, MTM2015-64382-P (MINECO/FEDER) and in part by Universidad Internacional de La Rioja (UNIR, http://www.unir.net), under the Plan Propio de Investigación, Desarrollo e Innovación [2015–2017]. Research group: Modelación matemática aplicada a la ingeniería (MOMAIN).

*Corresponding author
Email addresses: sergio.amat@upct.es (Sergio Amat), pedrojesus.blazquez@unir.net (Pedro J. Blázquez), sonia.busquier@upct.es (Sonia Busquier), concepcion.bermudez@upct.es (Concepción Bermúdez)