Gaussian quadrature rules for C^1 quintic splines with uniform knot vectors

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PII: S0377-0427(17)30087-0
DOI: http://dx.doi.org/10.1016/j.cam.2017.02.022
Reference: CAM 11024

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 24 November 2016
Revised date: 2 February 2017

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Abstract

We provide explicit quadrature rules for spaces of C^1 quintic splines with uniform knot sequences over finite domains. The quadrature nodes and weights are derived via an explicit recursion that avoids numerical solvers. Each rule is optimal, that is, requires the minimal number of nodes, for a given function space. For each of n subintervals, generically, only two nodes are required which reduces the evaluation cost by $2/3$ when compared to the classical Gaussian quadrature for polynomials over each knot span. Numerical experiments show fast convergence, as n grows, to the "two-third" quadrature rule of Hughes et al. [23] for infinite domains.

Keywords: Gaussian quadrature, quintic splines, Peano kernel, B-splines, C^1 continuity, quadrature for isogeometric analysis

1. Introduction

Numerical quadrature has been of interest for decades due to its wide applicability in many fields spanning collocation methods [37], integral equations [4], finite elements methods [38] and most recently, isogeometric analysis [13]. Numerical quadrature is an important tool for high-speed solution frameworks [12, 16] as it is computationally cheap and robust when compared to analytic integration methods [20].

A quadrature rule, or in short a quadrature, is said to be an *m-point rule*, if m evaluations of a function f are needed to approximate its weighted integral

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