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On block diagonal and block triangular iterative schemes and preconditioners for stabilized saddle point problems

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Abstract. We review the use of block diagonal and block lower/upper triangular splittings for constructing iterative methods and preconditioners for solving stabilized saddle point problems. We introduce new variants of these splittings and obtain new results on the convergence of the associated stationary iterations and new bounds on the eigenvalues of the corresponding preconditioned matrices. We further consider inexact versions as preconditioners for flexible Krylov subspace methods, and show experimentally that our techniques can be highly effective for solving linear systems of saddle point type arising from stabilized finite element discretizations of two model problems, one from incompressible fluid mechanics and the other from magnetostatics.

Keywords: iterative methods; convergence; saddle point problems; preconditioning.

AMS Subject Classification: 65F10.

1 Introduction

Consider a stabilized saddle point problem of the form

$$\mathcal{A}u \equiv \begin{pmatrix} A & B \\ -B^T & C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ -q \end{pmatrix} \equiv c, \tag{1}$$

where $A \in \mathbb{R}^{m\times m}$ is symmetric positive definite, $B \in \mathbb{R}^{m\times n}$ is of full column rank and C is symmetric positive semidefinite. We are especially interested in the case $C \neq 0$, although our theory also applies to the (non-stabilized) case where $C = 0$. Under these assumptions, (1) admits a unique solution; see, e.g., [4, Lemma 1.1].

Large linear systems of the form (1) arise in several areas of computational science and engineering, including computational fluid dynamics, constrained optimization, regularized least-squares problems, geomechanics, electromagnetics, and so on; see, e.g., [5, 6, 16, 17] and references therein for further details.

Over the years, a number of methods have been proposed in the literature for solving (1): we refer the reader to the survey [5] for developments up to about 2005. More recently, several authors have studied different variants of the classical Uzawa method, mostly for the case $C = 0$; see, for instance [27, 29, 31] and references therein. Moreover, several new preconditioners for Krylov subspace methods have been introduced in recent years, For example, Cao et al. [9] have studied the performance of a preconditioner obtained based on shift-splitting of the saddle point coefficient matrix. Then, Chen and Ma [12] have proposed a general class of preconditioners which incorporates as a special case the preconditioner given in [9]. We mention that the results in [9, 12] have been derived for the case that