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A filled function method for minimizing control variation in constrained discrete-time optimal control problems

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HIGHLIGHTS

- We present a filled function method for discrete-time optimal control problems.
- The cost function is the sum of terminal cost and the variation of control signal.
- The control problem is formulated as a non-smooth constrained optimization problem.
- Two examples are provided to demonstrate the feasibility of the method.

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ABSTRACT

This paper considers a discrete-time optimal control problem subject to terminal state constraints and all-time-step inequality constraints, where the cost function involves a terminal cost, a summation cost and a penalty on the change of the control action. The variation of the control signal and the all-time-step constraints are non-smooth functions. Thus, this optimal control problem is formulated as a non-smooth constrained optimization problem. However, it is nonconvex and hence it may have many local minimum points. Thus, a filled function method is introduced in conjunction with local optimization techniques to solve this non-smooth and nonconvex constrained optimization problem. For illustration, two numerical examples are presented and solved using the proposed approach.

1. Introduction

Optimal control has been an area of active research over several decades and it has found many real world applications in disciplines ranging from managing sciences to engineering. There are many papers and books being devoted to optimal control and its application, See, for example, [1–13], and references therein. Most of these optimal control problems are governed by continuous time systems. In this paper, our concern is on discrete-time optimal control problems. These problems are normally governed by systems of difference equations, under which a cost function is to be minimized subject to various constraints on the state and/or control variables [4,14,15]. They arise naturally in diverse disciplines, such as economics, biomedicine and engineering. In the existing literature, the cost function of a discrete-time optimal control problem is usually a function of the final state reached by the system and/or a summation term involving the state and

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control variables at each time point. The cost of the change of the control signal is ignored. However, for a real practical system, there is a cost for any fluctuation in the control signal. In particular, a large fluctuation is highly undesirable.

In this paper, we consider, as in [14], a general discrete-time optimal control problem subject to terminal state constraints and all-time-step inequality constraints, where the cost function involves a terminal cost, a summation cost and a penalty on the change of the control action. The variation term in the cost function is the summation of non-smooth absolute value functions. For the all-time-step inequality constraints, if the number of time steps is large, the number of the inequality constraints resulting from the all-time-step inequality constraints will also be large. Thus, these all-time-step inequality constraints are transcribed as equality constraints involving a summation of non-smooth maximum value functions. In passing, it is worth noting that these all-time-step inequality constraints could only be dealt with by using the exact penalty function method (see, for example, [16,17]). Now, this discrete-time optimal control problem is transcribed into an equivalent optimization problem subject to equality constraints involving a summation of non-smooth maximum value functions, where its cost function contains a term penalizing the variation of the control action which is non-smooth. Standard gradient-based optimization algorithms cannot be directly applied to solve it. In [14], novel smooth approximation techniques are applied to approximate the problem as a smooth constrained optimization problem. Then, gradient based optimization method can be developed to solve this approximate smooth constrained optimization problem. However, these problems (the original non-smooth problem and the smooth approximation problem) are non-convex. The gradient based methods, if applicable, could only find local optimal solutions but these non-convex, non-smooth constrained optimization problems may have many local minimum points. Furthermore, some of the cost values of these local solutions may be far away from the true global cost value. Thus, effective global algorithms are required to tackle this class of constrained optimization problems.

The filled function method, initially proposed in [18] for smooth optimization, and later on, being extended in various ways to solve different problems, which includes non-smooth optimization problems (see, for example, [19–21]), is one of the effective and practical global optimization methods. The essence of the method is to modify the objective function as a filled function, and then to find another smaller local minimum point gradually by optimizing the filled function constructed on the minimum point obtained in previous iteration. The filled function method is composed of a sequence of cycles, and each cycle has two stages: a local minimization stage and a filling stage. The two stages are carried out alternatively until a global minimum point is found. For more detailed discussions on various existing results available in the literature, see, for example, [5,22–25] and the references therein. Note that there are many other global optimization methods available in the literature, such as those reported in [26,27] and references therein.

Due to the promise for being able to find a global optimal solution, the filled function method has been applied to the study of various optimal control problems. For example, a global optimization method is developed in [6] to solve a class of impulsive optimal control problems, where a two-parameter filled function is introduced to escape from the local minimum point. However, for the filled function introduced in [6], it is required to assume that the objective function has only a finite number of local minimum points and there exists a minimum point on the line joining the prefixed point and the current local minimum point. Furthermore, the parameters are restricted by the minimal radius of the local minimum point domain. These features can affect the performance in computation. In [28], the aim is to find an optimal design of IIR filters, where a combined method is developed based on the constraint transcription [13] and a filled function method to find a global optimal solution. This filled function has some improved features over the one introduced in [6] and, in particular, the parameters involved are easier to be adjusted. In [7], the continuous filled function concept is extended to solve optimal discrete-valued control problems. In [29], the design of broadband beamformers with low complexity is formulated as a bi-objective integer programming problem, where the coefficients of the filters are expressed as sums of signed powers-of-two terms. Then, an approach based on a discrete filled function is proposed to obtain the global optimal design. In [8], an optimal control problem governed by switched systems with a continuous-time inequality constraint is considered. It is formulated as a bi-level optimization problem. A local optimization method based on the modified BFGS update is combined with a filled function method to develop a global optimization method to solve this bi-level optimization problem.

Inspired by the works done in [20,21,28,29], we will derive a new filled function method for solving the class of constrained discrete-time optimal control problems with a penalty on the change of the control action that is considered in this paper.

The remainder of the paper is organized as follows. Following this introduction, the non-smooth constrained optimization problem transformation is given in Section 2. Then in Section 3, an improved definition of filled function for non-smooth constrained optimization problems is proposed. A new two-parameter filled function together with the solution algorithm is suggested in Section 4. Two examples are computed in Section 5 to demonstrate the feasibility and versatility of the method. Finally, some suggestions and conclusions are given in Section 6.

2. Problem transformation

Consider a discrete-time dynamic system described by the following system of difference equations:

$$x(k+1) = f(k, x(k), u(k)), \quad k = 0, 1, \ldots, M-1, \tag{1}$$

with the initial condition

$$x(0) = x^0, \tag{2}$$