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Weak rate of convergence of the Euler-Maruyama scheme for stochastic differential equations with non-regular drift

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Abstract

We consider an Euler-Maruyama type approximation method for a stochastic differential equation (SDE) with a non-regular drift and regular diffusion coefficient. The method regularizes the drift coefficient within a certain class of functions and then the Euler-Maruyama scheme for the regularized scheme is used as an approximation. This methodology gives two errors. The first one is the error of regularization of the drift coefficient within a given class of parametrized functions. The second one is the error of the regularized Euler-Maruyama scheme. After an optimization procedure with respect to the parameters we obtain various rates, which improve other known results.

Keywords. Stochastic differential equation, Euler-Maruyama scheme, discontinuous drift, weak rate of convergence, Malliavin calculus.
MSC (2010): Primary 65C30, 60H10

1 Introduction

The Euler-Maruyama scheme is a simple and efficient numerical scheme to simulate solutions of multi-dimensional stochastic differential equations (SDE's) such as

$$X_t = x + \int_0^t \sigma(s, X_s)\,\mathrm{d}B_s + \int_0^t b(s, X_s)\,\mathrm{d}s, \tag{1}$$

where B is a multi-dimensional Brownian motion. In many situations, one is interested in computing quantities of the type $\mathbb{E}[f(X_T)]$ for some $T > 0$ and $f \in \mathfrak{F}$ where $\mathfrak{F}$ is a class of functions. For a fixed number n of steps, consider n independent Gaussian random vectors

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