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High-order collocation methods for nonlinear delay integral equation

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Abstract

The classical collocation methods based on piecewise polynomials have been studied for delay Volterra integral equations of the second-kind in [5]. These collocation methods have uniform order m for any choice of the collocation parameters and can achieve local superconvergence in the grid points by choosing the suitable collocation parameters. In this paper with the aim of increasing the order of classical collocation methods, we use a general class of multistep methods based on Hermite collocation methods and prove that this numerical method has uniform order $2m + 2r$ for r previous time steps and m collocation points. Some numerical examples are given to show the validity of the presented method and to confirm our theoretical results.

AMS subject classifications: 65R20; 45D05; 34K06.

Keywords: Delay integral equations; Multistep collocation methods; Hermite collocation methods; Convergence analysis.

1 Introduction

Models involving retarded integral, ordinary and partial differential equations with both discrete and distributed delays are quite frequent in mathematical biology. Introducing delays in the models can be necessary for simulating the evolution phenomena in many branches of medicine and biology. Introduction of delays allowed to improve models by taking into account important aspects previously neglected and to face more complicated phenomena based on feedback control. It was shown that arising of periodic hematological diseases can be caused by anormalities in the feedback mechanism which regulate blood-cell numbers and, under appropriate conditions, this feedback mechanism can produce aperiodic irregular (chaotic) fluctuations. Recently, in the interaction analysis between cardiovascular and respiratory function, clever models have been considered that take into account the time necessary for tissue venous blood to reach the lungs and viceversa [2]. In biomathematical literature there are plenty examples where the presence of delays makes the mathematical models much more reliable and consistent with the real phenomena and the laboratory observations. Actually the dynamics of equations including retarded arguments is much richer and this makes the models more realistic for simulation. At the same time, the equations with retarded arguments become more and more complicated to be analyzed and the existence and uniqueness of the solution as well as important features such as oscillation and asymptotic behavior are still open problems in many cases. Finding accurate