Accepted Manuscript

A combination of multiscale time integrator and two-scale formulation for the nonlinear Schrödinger equation with wave operator

Xiaofei Zhao

PII: S0377-0427(17)30306-0
DOI: http://dx.doi.org/10.1016/j.cam.2017.06.006
Reference: CAM 11184

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 21 December 2016
Revised date : 27 April 2017

Please cite this article as: X. Zhao, A combination of multiscale time integrator and two-scale formulation for the nonlinear Schrödinger equation with wave operator, *Journal of Computational and Applied Mathematics* (2017), http://dx.doi.org/10.1016/j.cam.2017.06.006

A combination of multiscale time integrator and two-scale formulation for the nonlinear Schrödinger equation with wave operator

Xiaofei Zhao[a,b]

[a] INRIA-Rennes Bretagne Atlantique, IPSO Project, France
[b] IRMAR, Université de Rennes 1, Rennes, France

Abstract

In this paper, we consider the nonlinear Schrödinger equation with wave operator (NLSW), which contains a dimensionless parameter $0 < \varepsilon \leq 1$. As $0 < \varepsilon \ll 1$, the solution of the NLSW propagates fast waves in time with wavelength $O(\varepsilon^2)$ and the problem becomes highly oscillatory in time. The oscillations come from two parts. One part is from the equation and another part is from the initial data. For the ill-prepared initial data case as described in [2] which brings inconsistency in the limit regime, standard numerical methods have strong convergence order reduction in time when ε becomes small. We review two existing methods to solve the NLSW: an exponential integrator and a two-scale method. We comment on their order reduction issues. Then we derive a multiscale decomposition two-scale method for solving the NLSW by first performing a multiscale decomposition on the NLSW which decomposes it into a well-behaved part and an energy-unbounded part, and then applying an exponential integrator for the well-behaved part and a two-scale approach for the energy-unbounded part. Numerical experiments are conducted to test the proposed method which shows uniform second order accuracy without significant order reduction for all $0 < \varepsilon \leq 1$. Comparisons are made with the existing methods.

Keywords: nonlinear Schrödinger equation with wave operator, highly oscillatory, ill-prepared initial data, multiscale decomposition, two-scale formulation, uniformly accurate, order reduction

1. Introduction

In this paper, we consider the following nonlinear Schrödinger equation with wave operator (NLSW) in d-dimensions ($d = 1, 2, 3$) [1, 2, 29, 19]:

$$\begin{cases} 2i\partial_t u^\varepsilon(\mathbf{x},t) - \varepsilon^2\partial_{tt}u^\varepsilon(\mathbf{x},t) + \Delta u^\varepsilon(\mathbf{x},t) + f\left(|u^\varepsilon(\mathbf{x},t)|^2\right)u^\varepsilon(\mathbf{x},t) = 0, \quad \mathbf{x}\in\mathbb{R}^d,\ t>0, \\ u^\varepsilon(\mathbf{x},0) = \varphi_1(\mathbf{x}), \quad \partial_t u^\varepsilon(\mathbf{x},0) = \varphi_2(\mathbf{x}), \quad \mathbf{x}\in\mathbb{R}^d. \end{cases} \tag{1.1}$$

Here t is time, $\mathbf{x}$ is the spatial coordinate, $u^\varepsilon := u^\varepsilon(\mathbf{x},t)$ is the unknown complex-valued scalar field, $0 < \varepsilon \leq 1$ is a parameter, φ_1 and φ_2 are two given initial functions which are uniformly bounded for all $0 < \varepsilon \leq 1$, and $f(\cdot) : \mathbb{R} \to \mathbb{R}$ is a given nonlinearity independent of ε.

The above NLSW comes widely from both mathematical analysis and physics applications. On the mathematical side, in the theoretical analysis of the nonrelativistic limit of the nonlinear Klein-Gordon equation, the NLSW (1.1) has been considered as an intermediate limit model which has bounded energy as $\varepsilon \to 0$ [24, 25, 23, 28]. On the physics applications side, (1.1) has been derived as the model for the envelope approximation of Langmuir turbulence [5, 13] or the modulated planar pulse approximation of the sine-Gordon equation for light bullets [7, 30]. In these cases, the ε represents a dimensionless parameter

Email address: zhxfnus@gmail.com (Xiaofei Zhao)