Accepted Manuscript

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PII: S0377-0427(17)30004-3
DOI: http://dx.doi.org/10.1016/j.cam.2017.01.001
Reference: CAM 10965

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 24 April 2016
Revised date: 31 December 2016

Some results on upper bounds for the variance of functions of the residual life random variables

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Abstract

As a measure of maximum dispersion from the mean, upper bounds on variance have applications in all areas of theoretical and applied mathematical sciences. In this paper, we obtain an upper bound for the variance of a function of the residual life random variable X_t. Since one of the most important types of system structures is the parallel structure, we give an upper bound for the variance of a function of this system consisting of n identical and independent components, under the condition that, at time t, $n-r+1$, $r=1,\ldots,n$ of its components are still working. Here we characterize the Pareto distribution through Cauchy's functional equation for mean residual life. It is shown that the underlying distribution function F can be recovered from the proposed mean and variance residual life function of the system for $r=1$. Moreover, we see that the variance residual lifetime of the components of the system is not necessarily a decreasing function of r and increasing of n for $r=1$, unlike their mean residual lifetime. As an application, the variance of $X_{F^{-1}(p_0)}$ for all $p_0 \in [0,1)$ is investigated and also a real data analysis is presented.

Keywords: Characterization, Mean residual life function, Variance residual life function, Parallel systems, Variance bound.
Mathematical Subject Classification: Primary 60E15; Secondary 62N05.

1 Introduction

The mean residual life (MRL) function has been widely used in reliability. For example, it is used to design burn-in programs, plan spare provision, and formulate warranty policies. Let X be a non-negative random variable having absolutely continuous distribution function $F(t)$ and survival function $\overline{F}(t) = 1 - F(t)$ and probability density function $f(t)$. Then the hazard rate function of X is defined as $r(t) = -\frac{d}{dt}\log\overline{F}(t) = \frac{f(t)}{\overline{F}(t)}$.

An useful reliability measure of X is mean residual life, which is defined as expectation of the residual life random variable $X_t = (X - t \mid X > t)$, given by

$$m(t) = \frac{1}{\overline{F}(t)}\int_t^{\infty}\overline{F}(x)dx. \tag{1}$$

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