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Regularization of the Cauchy problem for the Helmholtz equation by using Meyer wavelet

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Abstract

In this paper, we investigate a Cauchy problem associated with Helmholtz- type equation in an infinite "strip". This is a classical severely ill-posed problem, i.e., the solution (if it exsits) does not depend continuously on the data (or Cauchy data), a small perturbation in the data can cause a dramatically large error in the solution for $0 < x \leq 1$. The stability of the solution is restored by using a wavelet regularization method. Moreover, some sharp stable estimates between the exact solution and it's approximation in $H^r(\mathbb{R})$-norm is also provided and the numerical examples show that the method works effectively.

Keywords: Cauchy problem; Helmholtz equation; Regularization; Meyer wavelet; Multiresolution analysis.

2010 Mathematics Subject Classification: 35J05, 65F22, 65T60, 47J06.

1 Introduction

The Helmholtz equation is a special kind of elliptic equation and is specially important in some practical physical applications. It is often used to describe the vibration of a structure [1], the acoustic cavity problem [2], the radiation wave [8], the Poisson-Boltzmann equation [16], etc. For more information about the Cauchy problem of Helmholtz equations one can refer to [18, 28].

The Cauchy problem of an elliptic equation is well known to be ill-posed in the sense of Hadamard. The direct problem for the Helmholtz equation, i.e., Dirichlet, Neumann or mixed boundary value problems have been studied extensively in the past century. However, in some practical problems, the boundary data on the whole boundary can not be obtained. For computational aspects, the readers can consult Hào and Lesnic [9], Reinhardt et al [29], Cheng and Yamamoto [3] and Hon and Wei [11]. For theoritical aspects, the reader can refer to Xiong [33], Xiong and Fu [34] and Qian et al [25].

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