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Second order accuracy finite difference methods for space-fractional partial differential equations[☆]

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Abstract

Space-fractional partial differential equations are used for simulations of, for example, diffusion of radioactive materials, and financial and other models, which are characterized by heavy-tailed distributions. A number of first order accuracy finite difference methods have been proposed. In the present paper, we introduce second order accuracy finite difference methods with Dirichlet boundary conditions. These methods have a parameter in these schemes, and the parameter stabilizes the schemes. This means that there exist various schemes with second order accuracy, but the stability of each scheme is different. In the present paper, we introduce the most stable scheme for any fractional calculus order by choosing the optimal parameter. In addition, we describe a phenomenon whereby the expected accuracy cannot be obtained if the analytical solution can be expanded to a series having less than second order around boundaries. This also happens in both existing methods and the proposed methods. In the present paper, we develop the stability conditions for the proposed schemes, and numerical examples of second order accuracy and accuracy decay are shown.

Keywords: fractional calculus, space-fractional partial differential equations, finite difference method, difference formula, boundary problem, Dirichlet boundary condition, partial differential equations

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