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Optimal control of water flooding reservoir using proper orthogonal decomposition

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HIGHLIGHTS

- A reduced-order optimal control methodology for water flooding reservoir is proposed.
- With the reduced-order optimal control methodology, only a set of POD coefficients are considered as optimization parameters.
- Compared with the adjoint-based gradient methodology with full-order model, the new methodology performs much better in calculation speed, and its advantage significantly increases as the number of grid blocks increases.

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ABSTRACT

Optimal control of water flooding reservoir is performed to calculate the optimal set of well controls to maximize economic profitability. However, calculating the optimal well controls requires major computational resources, which restricts its implementation in actual reservoir production. In this paper a reduced-order optimal control methodology mainly based on POD (Proper Orthogonal Decomposition) is proposed to optimize the operation of wells in water flooding reservoir. The methodology is decomposed into the 'offline' and 'online' calculations. The offline calculation consists in determining a reduced set of POD basis from several 'snapshots' obtained using reservoir simulator over a pre-determined set of time instants and well control parameters. In the online calculation, knowing the current economic parameters, the optimal well controls are determined only using a nonlinear programming method and POD reconstruction. The methodology is approved on a small scale, two-dimensional, water flooding reservoir model. The results show that the NPV obtained by the reduced-order optimal control methodology is approached to within 99% of the NPV obtained by the adjoint-gradient based methodology, besides, it is quite fast, where the achieved increase in calculation speed is several dozens of times, and its advantage of calculation speed significantly increases as the number of grid blocks increases.

1. Introduction

Optimal control of water flooding reservoir concerns adjustments of well controls, for example, fluid flow rates and bottom hole pressures (BHP) in the injection and production wells aiming to maximize some economic objectives (such as net-present-value (NPV)) or oil recovery over the producing life of a certain reservoir model. Optimal control of reservoir production is one of the two main steps of closed-loop reservoir management [1–4], the other step is to update the

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parameters of the reservoir model with history-matching, in this process, the discrepancy between the actual production data and the production data from the reservoir model is minimized. In this paper we will only concentrate on the optimal control aspect but not address the history-matching problem.

Optimal control of water flooding reservoir production is a large-scale optimization problem accompanied with a great number of control variables and grid blocks, the relationship between well controls and objective function is governed by a set of nonlinear partial differential equations. Several direct optimization methodologies [1–3,5–12] including Ensemble-based optimization methodologies, optimization based on Quadratic Interpolation and adjoint-gradient based methodologies have been proposed and applied to determine the optimal well controls of water flooding reservoir. The most attractive and efficient ways by far are through the use of adjoint-gradient based methodology [1,5–11] in which obtaining each new estimate of well controls can be achieved by means of calculating gradients of an objective function with respect to the well controls at every time step. However, calculating gradients requires several passes of forward full-order simulations governed by large amounts of partial differential equations of the reservoir model and backward simulations of an adjoint system of equations, in which an N-dimensional adjoint vector is required to be determined in solving every adjoint equation and several N-dimensional matrixes must be calculated to obtain every element of adjoint vector, where N denotes the number of grid blocks of reservoir model, it is a great burden to directly numerically calculate the optimal control solutions with the current speed and storage space of computer. For a typical reservoir simulation model which contains $10^6 - 10^8$ grid blocks, the computational times for determining the optimal well controls are always in the order of several days or even weeks although the requirement for the performance of computer is satisfied. What is more, the producing life and some economic factors such as the oil price and the annual discount rate used to calculate the NPV are always not constant because of the market fluctuation factors, which means that the optimal well controls determined at the beginning are not applicable all the while, thus the optimal well controls are required to be updated in real-time. Therefore, an accurate, efficient computational optimal control model that can be solved sufficiently rapidly is necessary to be developed.

Reduced-order models are representative of large scale systems that result from discretization of partial differential equations. Most reduced-order models are obtained by a projection framework, that is, the governing equations of large scale are projected onto the subspace spanned by a reduced set of basis, where the basis represents the dynamic characteristics of the original large-scale physical system. For the basis, a widely applied one is the POD [13] basis which is determined based on a large number of state snapshots obtained through actual experimental data or high fidelity numerical simulations over a desired range of control parameters. The significant advantage of the POD basis is its energetic optimality: few modes may contain almost the total energy. Therefore, POD has been applied in many different settings with considerable success, including unsteady fluid dynamics [14–16], heat transfer [17], structural dynamics [18,19] circuit design [20], inverse problems [21–23] and optimal control problems [24–30].

For the applications of POD in the context of optimal control problems, for example, in Tallet et al. [27], POD was used to determine in real-time the temperature distribution of thermal comfort in a room. The offline step consists in selecting snapshots (velocity and temperature fields) over several air inlet velocity and temperature with CFD numerical simulations. This offline step enables to obtain thereafter the POD reduced basis. In the online step, knowing the inlet and outdoor temperatures, the desired temperature distribution in the occupation zone is obtained in real-time using an optimization algorithm and POD reconstruction. Similarly, My-Ha et al. [28] used POD to determine in real-time the depths and strengths of water bubbles that beat match a desired free surface shape. In Guha et al. [29], for generation of the POD basis, the electromagnetic model is first simulated with a sinusoidal current of constant magnitude and frequency, and temperature profiles at each time-step are stored as snapshots, then applying the Hamilton–Jacobi–Bellman formalism to the POD based reduced-order model, an optimal control input current for the induction heating system is obtained in real-time for transferring the temperature profile over an axi-symmetric work piece from any initial temperature profile to any desired temperature profile. For optimal control of water flooding reservoir, few literatures are reported, as a representative application, Doren et al. [30] used POD to speed up the water flooding optimization procedure. They developed a nested approach that an inner loop was used to perform the optimization in reduced-order space and the approximate results from the inner loop were corrected by an outer loop using the full-order model, which strongly reduces the number of underlying governing equations that need to be solved during the Newton–Raphson iterations. Although the resulting reduction in CPU time was just 35%, the results indicated that the dynamic characteristics of water flooding with a fixed well configuration is accurately governed by a set of low-order equations. How to develop a more practically available approach using POD to achieve computational speed-ups for water flooding reservoir optimization needs further research.

This paper proposes a reduced-order optimal control methodology for water flooding reservoir with the combination of reduced-order models and optimal control model, the relationship between the control variables and objective function is connected by analytic function but not governed by a set of nonlinear partial differential equations, only a small amount of POD coefficients are considered as optimization variables, it will be shown that the optimal control of water flooding reservoir is just a common nonlinear programming problem, which considerably reduces the complexity and the amount of calculation.

This paper proceeds as follows: The general dynamical system framework with full-order model and the considered optimal control problem for water flooding reservoir are respectively presented in Sections 2 and 3. Then the fundaments of POD and the reduced-order model of water flooding reservoir are shown in Section 4. Thereafter, the design for the reduced-order optimal control methodology are described in detail in Section 5. Finally, in Section 6, we show an example