Accepted Manuscript

Application of operational matrices for solving system of linear Stratonovich Volterra integral equation

Farshid Mirzaee, Nasrin Samadyar

PII: S0377-0427(17)30065-1
DOI: http://dx.doi.org/10.1016/j.cam.2017.02.007
Reference: CAM 11009

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 16 August 2016
Revised date: 1 January 2017

Application of operational matrices for solving system of linear Stratonovich Volterra integral equation

Farshid Mirzaee [1] **, Nasrin Samadyar**

Faculty of Mathematical Sciences and Statistics, Malayer University, P. O. Box 65719-95863, Malayer, Iran

Abstract

In this paper, we give a brief review on Block pulse functions (BPFs) and obtain operational matrix and stochastic operational matrix of integration based on BPFs. Then these operational matrices are used to solve system of linear Stratonovich Volterra integral equations. By applying proposed method, the system of linear Stratonovich Volterra integral equations reduce to system of linear algebraic equations which can be solved by a convenient numerical method. Also, the error analysis is proved under several mild conditions. We show the rate of convergence is $O(h)$. Finally, by applying this method on two examples, we demonstrate accuracy and efficiency of the proposed method. All of the numerical calculation are performed on computer using a program written in MATLAB.

Keywords: Linear Stratonovich Volterra integral equations; Stochastic operational matrix; Computational methods for stochastic equations; Brownian motion process; Block pulse functions; Error analysis.

AMS Subject Classification: 65C30, 60H20, 60H35, 60J65.

1 Introduction

Stochastic processes were studied harshly in the late 19th century for the first time to aid in understanding financial markets and Brownian motion. Stochastic integrals are divided into Itô integral and Stratonovich integral. The Itô integral named after Kiyoshi Itô, whereas the Stratonovich integral developed simultaneously by R. L. Stratonovich [1] and D. L. Fisk [2]. In stochastic processes Stratonovich integral and Itô integral are two stochastic integral that Itô integral is usually used in applied mathematics and the Stratonovich integral is frequently used in physics.

Unlike the Itô calculus, Stratonovich integrals are defined such that the chain rule of ordinary calculus holds. This calculus is designed in such a way that its basic rules, such as the chain rule and integration by parts are the same as in the standard calculus. For this reason, stochastic integrals in applications are often taken in the sense of Stratonovich calculus. Although the rules of manipulations are the same, the calculus are still very different. The processes need to be adapted, just as in Itô calculus. Since Stratonovich stochastic integrals can be reduced to Itô integrals, the standard stochastic differential equation (SDE) theory can be used for Stratonovich stochastic differential equations.

The Stratonovich integral can be defined similar to the Riemann integral, that is as a limit of Riemann sums. Because of the difficulty and complexity, often we do not solve Stratonovich Volterra integral equation analytically and we try solve it numerically. Various numerical methods are used to solve stochastic integral equations [3-13]. Recently Mirzaee et al. have been applied Bernoulli polynomials for solving nonlinear Stratonovich Volterra integral equations [14]. Also Mirzaee and Hadadiyan by applying modification of hat functions, get approximate solution of nonlinear Stratonovich Volterra integral equation[15]. But, there are still very few papers discussing the numerical solutions of Stratonovich Volterra integral equations.

In this paper by applying Block pulse functions, we get an approximate solution for linear system of Stratonovich Volterra integral equation as follows

$$\mathbf{f}(x) = \mathbf{g}(x) + \int_0^x \mathbf{k1}(x,y)\mathbf{f}(y)dy + \int_0^x \mathbf{k2}(x,y)\mathbf{f}(y) \circ dB(y), \quad (1)$$

[1] Corresponding author. Tel.: +98 81 32355466; Fax: +98 81 32355466.

E-mail addresses: f.mirzaee@malayeru.ac.ir, fa_mirzaee@yahoo.com (F. Mirzaee)
nasrin.samadyar@stu.malayeru.ac.ir (N. Samadyar)