Accepted Manuscript

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Zhongdi Cen, Anbo Le, Aimin Xu

PII: S0377-0427(17)30067-5
DOI: http://dx.doi.org/10.1016/j.cam.2017.02.009
Reference: CAM 11011

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 9 October 2015
Revised date: 8 February 2017

Parameter-uniform hybrid difference scheme for solutions and derivatives in singularly perturbed initial value problems

Zhongdi Cen[1], Anbo Le, Aimin Xu

Institute of Mathematics, Zhejiang Wanli University,
Ningbo 315100, Zhejiang, P.R. China

Abstract. A second-order initial value problem with a small parameter multiplying the first- and second-order derivatives is considered. The precise knowledge about the behavior of the exact solution is analyzed. Based on the behavior of the exact solution, a hybrid finite difference scheme on a Shishkin mesh is proposed, which is a combination of the second-order difference scheme on the fine mesh and the modified midpoint upwind scheme on the coarse mesh. By applying the truncation error estimate techniques and a difference analogue of Gronwall's inequality we prove that the scheme is almost second-order convergent for numerical solutions and scaled numerical derivatives. Numerical experiments support these theoretical results and indicate that the estimates are sharp.

Key words. Singular perturbation; difference scheme; initial value problem; Shishkin mesh; uniform convergence

AMS subject classifications. Primary 65L10, Secondary 65L12, 65L50

1 Introduction

This paper is prompted by a publication [1] where the authors consider the following singularly perturbed initial value problem:

$$\begin{cases} Lu(x) \equiv \varepsilon^2 u''(x) + \varepsilon a(x) u'(x) + b(x)u(x) = f(x), & x \in \Omega = (0, T], \\ u(0) = A, \\ L_0 u(0) \equiv u'(0) = B/\varepsilon, \end{cases} \tag{1.1}$$

where $0 < \varepsilon \ll 1$ is a small parameter, A and B are given constants, $a(x), b(x)$ and $f(x)$ are sufficiently smooth functions satisfying $a(x) \geq \alpha > 0, \beta^* \geq b(x) \geq \beta > 0$. In general, the solution $u(x)$ has a boundary layer at $x = 0$ (see [1]). The class of

[1]Email: finite@zwu.edu.cn.