Accepted Manuscript

Analysis of a conforming finite element method for the Boussinesq problem with temperature-dependent parameters

Ricardo Oyarzúa, Paulo Zúñiga

PII: S0377-0427(17)30164-4
DOI: http://dx.doi.org/10.1016/j.cam.2017.04.009
Reference: CAM 11088

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 21 August 2015
Revised date: 27 February 2017

Analysis of a conforming finite element method for the Boussinesq problem with temperature-dependent parameters *

Ricardo Oyarzúa† and Paulo Zúñiga‡

Abstract

In this paper we analyze a conforming finite element method for the numerical simulation of non-isothermal incompressible fluid flows subject to a heat source modelled by a generalized Boussinesq problem with temperature-dependent parameters. We consider the standard velocity-pressure formulation for the fluid flow equations which is coupled with a primal-mixed scheme for the convection-diffusion equation modelling the temperature. In this way, the unknowns of the resulting formulation are given by the velocity, the pressure, the temperature, and the normal derivative of the latter on the boundary. Hence, assuming standard hypotheses on the discrete spaces, we prove existence and stability of solutions of the associated Galerkin scheme, and derive the corresponding Cea's estimate for small and smooth solutions. In particular, any pair of stable Stokes elements, such as Hood-Taylor elements, for the fluid flow variables, continuous piecewise polynomials of degree $\leq k+1$ for the temperature, and piecewise polynomials of degree $\leq k$ for the boundary unknown become feasible choices of finite element subspaces. Finally, we derive optimal a priori error estimates, and provide several numerical results illustrating the performance of the conforming method and confirming the theoretical rates of convergence.

Key words: generalized Boussinesq problem, conforming finite element method, Hood-Taylor element, temperature-dependent parameters, primal-mixed formulation

Mathematics Subject Classifications (2000): 65N30, 65N12, 65N15, 35Q79, 80A20, 76R05, 76D07

1 Introduction

This paper is concerned with the numerical approximation of the stationary generalized Boussinesq problem:

$$\begin{aligned} -\mathbf{div}\,(\nu(\theta)\nabla\mathbf{u}) + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p - \mathbf{g}\theta &= \mathbf{0} \quad \text{in} \quad \Omega,\\ \mathrm{div}\,\mathbf{u} &= 0 \quad \text{in} \quad \Omega,\\ \mathbf{u} &= \mathbf{0} \quad \text{on} \quad \Gamma,\\ -\mathrm{div}\,(\kappa(\theta)\nabla\theta) + \mathbf{u}\cdot\nabla\theta &= 0 \quad \text{in} \quad \Omega,\\ \theta &= \theta_D \quad \text{on} \quad \Gamma. \end{aligned} \tag{1.1}$$

*This research was partially supported by CONICYT-Chile through project Fondecyt 11121347, project Anillo ACT1118 (ANANUM); and by Universidad del Bío-Bío through DIUBB project GI 151408/VC.

†GIMNAP-Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile and CI^{2}MA, Universidad de Concepción, , Casilla 160-C, Concepción, Chile, email: royarzua@ubiobio.cl

‡GIMNAP-Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile, email: pzuniga@ubiobio.cl