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An Adaptive Huber Method for Nonlinear Systems of Volterra Integral Equations with Weakly Singular Kernels and Solutions

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Abstract

Numerical methods for solving nonlinear systems of weakly singular Volterra integral equations (VIEs) possessing weakly singular solutions appear almost nonexistent in the literature, except for a few treatments of single first kind Abel equations. To reduce this gap, an extension is presented, of the adaptive Huber method designed for VIEs with singular kernels such as $\mathscr{K}(t,\tau) = (t-\tau)^{-1/2}$ and $\mathscr{K}(t,\tau) = \exp[-\lambda(t-\tau)](t-\tau)^{-1/2}$ (where $\lambda \geq 0$) and a variety of nonsingular kernels. The method was thus far restricted to bounded solutions having at least two derivatives. Under a number of assumptions specified, the extension applies to solutions $U_\mu(t)$ that can be written as sums of singular components $c_\mu t^{-1/2}$ (with unknown coefficients c_μ), and nonsingular components $\overline{U}_\mu(t)$. In the solution process, factor $t^{-1/2}$ is handled analytically, whereas c_μ and $\overline{U}_\mu(t)$ are determined numerically. Computational experiments reveal that the extended method determines singular solutions equally well as the unextended method determined nonsingular solutions. The method is intended primarily for a class of VIEs encountered in electroanalytical chemistry, but it can also be of interest to other application areas.

Keywords: Volterra integral equations, weakly singular kernels, weakly singular solutions, adaptive methods, product-integration, computational electrochemistry

1. Introduction

The present study is inspired by the class of one-dimensional Volterra integral equations (VIEs) of the first and second kind, encountered in electroanalytical chemistry, and representing models of transient electrochemical experiments [1]. VIEs of this class typically involve integrals with weakly singular convolution

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