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On partially inexact HSS iteration methods for the complex symmetric linear systems in space fractional CNLS equations

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Abstract

The space fractional coupled nonlinear Schrödinger (CNLS) equations are discretized by an implicit conservative difference scheme with the fractional centered difference formula, which is unconditionally stable. The coefficient matrix of the discretized linear system is equal to the sum of a complex scaled identity matrix and a symmetric positive definite diagonal-plus-Toeplitz matrix. The Hermitian and skew-Hermitian splitting (HSS) method and the partially inexact HSS (PIHSS) method are employed to solve the discretized linear system. In the inner iteration processes of the HSS method, we only need to solve the linear sub-systems associated with the Hermitian part inexactly by the conjugate gradient (CG) method, resulting in PIHSS iteration method. Theoretical analyses show that both HSS and PIHSS methods are unconditionally convergent. Numerical examples are given to demonstrate the effectiveness of the HSS iteration and the PIHSS iteration.

Keywords: The space fractional Schrödinger equations; Hermitian and skew-Hermitian splitting; Inexact iterations; Conjugate gradient method; Convergence analysis

1 Introduction

In this paper, we consider the space fractional coupled nonlinear Schrödinger (CNLS) equations

$$\begin{cases} iu_t + \gamma(-\Delta)^{\frac{\alpha}{2}}u + \rho(|u|^2 + \beta|v|^2)u = 0, \\ iv_t + \gamma(-\Delta)^{\frac{\alpha}{2}}v + \rho(|v|^2 + \beta|u|^2)v = 0, \end{cases} \quad a \le x \le b, \ \ 0 < t \le T, \tag{1.1}$$

with the initial boundary value conditions

$$\begin{cases} u(x,0) = u_0(x), & v(x,0) = v_0(x), & a \le x \le b, \\ u(a,t) = u(b,t) = 0, & v(a,t) = v(b,t) = 0, & 0 \le t \le T, \end{cases}$$

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