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Multivariate European option pricing in a Markov-modulated Lévy framework

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ABSTRACT

This paper studies the pricing of some multivariate European options, namely Exchange options and Quanto options, when the risky assets involved are modelled by Markov-Modulated Lévy Processes (MMLPs). Pricing formulae are based upon the characteristic exponents by using the well known FFT methodology. We study these pricing issues both under a risk neutral martingale measure and the historical measure. The dependence between the asset's components is incorporated in the joint characteristic function of the MMLPs. As an example, we concentrate upon a regime-switching version of the model of Ballotta et al. (2015) in which the dependence structure is introduced in a flexible way. Several numerical examples are provided to illustrate our results.

1. Introduction

It has already been recognized for a long time that the classical Black–Scholes model has limitations when pricing options. However, models based on the exponential Lévy processes are not able as well to capture all empirical features of the market because of the independence and stationarity of the Lévy process increments, as pointed out e.g. in [1]. A natural way to relax the stationarity and independence of the increments is to allow the process to change its parameters at certain times to reflect e.g. the fact that there exist business cycles in the market. This leads to the so called regime-switching models where a Markov process serves to describe parameter changes.

The regime-switching framework has been largely developed in the literature, although most of the articles on the subject are dedicated to univariate options. The prices of the risky asset involved in univariate options are often modelled as the exponential of a Markov-modulated Brownian motion (MMBM). Buffington and Elliot [2] found the Black–Scholes equations in the two states regime-switching setting and used it to price vanilla and American options. In [3], a partial differential equations approach is developed to provide an algorithm allowing to price numerically Asian options and lookback options. More recently, Zhu et al. [4] deal with the Fourier Transform method to derive a closed-form formula to price vanilla options in the MMBM setting. We refer to Elliott et al. [5] for a good overview of the literature and financial applications of regime-switching models. In this important paper, the authors introduce moreover a regime-switching Esscher transform in order to determine an equivalent martingale measure, since the market described by a MMBM is incomplete in general.

Markov-modulated Lévy processes (MMLPs) have also been used to price univariate options. In [1], the authors consider the two state case and suppose that the risky asset is a Variance Gamma process in each phase, under a risk neutral

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measure. They calculate the corresponding characteristic function to estimate the parameters and price the options. Elliott and Osakwe [6] generalize this paper by considering a pure jump switching process with an arbitrary number of states. A more general approach is followed in [7], where the author prices vanilla and exotic options when the price of the risky asset is the exponential of a one-dimensional MMLP. A transition in the Markov process can also induce a jump in the asset price. The author starts under the risk neutral measure and employs the Carr–Madan formula (see [8]) to price vanilla options and gives numerical procedures to deal with exotic options.

Regime-switching models further have shown to be useful in the study of currency options. In [9], the authors perform the valuation of European and American currency options in the context of MMBMs for the FX rate. They start from the real world measure and then employ the Esscher change of measure introduced in [5] to determine a risk neutral measure and explicit pricing formulae under that measure. For European option pricing, Bo et al. [10] generalize this model by adding jumps with lognormal distributions.

The main focus of this paper is to concentrate on multivariate options in a regime-switching framework. In this context, Yoon et al. [11] start from a risk neutral measure and propose an analytic valuation method based on the occupation times to price European-type multivariate contingent claims. In particular, they price quanto options and exchange options when the risky asset is modelled by a two-dimensional geometric MMBM with two phases. Chen et al. [12] concentrate upon European quanto options when the forward interest rates are supposed to follow a regime-switching HJM model, the stock prices by a regime-switching jump–diffusion and the spot FX rate is supposed to be into the Black–Scholes setting. They adapt the approach in [5] in order to obtain a risk neutral measure. In these papers, dependence between the different assets is easily described by the correlation between the Brownian motions showing up in the MMBM parts.

In this paper, we study three types of European options under the hypothesis that the risky assets involved are general Markov-Modulated Lévy Processes (MMLPs) modulated by a Markov chain with an arbitrary number of states. We begin by revisiting the pricing of European call options, for which we give some original examples in order to illustrate the large variety of Markov models that can be used to describe a given regime-switching situation, and to show how these models can lead to different option prices. Next, we apply the introduced methods into the pricing of European exchange and quanto options, which is the main contribution of this paper. As these options deal with two-dimensional MMLPs, the dependence between the two components has to be taken into account. We first provide pricing formulae that are valid whatever the correlation structure between the components. Then, we generalize the model introduced in [13] to the regime-switching case. As in [14], the dependence structure is then incorporated in a flexible way, and the pricing formulae can be easily implemented. We choose the Variance Gamma case to provide numerical illustrations. We derive the option prices by using the Fast Fourier Transform as presented in [8], and therefore calculate the necessary characteristic exponents.

As usual, the pricing analysis is performed under a risk neutral measure. However it appears sometimes more interesting to start from the historical probability, e.g. for asset–liability management reasons or in order to estimate the Markov chain parameters (see e.g. [15]). The last section of this paper is therefore devoted to the derivation of the pricing formulae starting from a real world model. Since the financial market is incomplete, the risk-neutral measure is not uniquely determined. We follow the Esscher transform approach, introduced in [16] and adapted to the regime-switching case in [5], to define a risk neutral measure. We state conditions that ensure the existence of such a measure in our framework and then we give the link with the pricing formulae obtained in the preceding sections.

The paper is organized as follows: Section 2 is devoted to vanilla options. Exchange options and quanto options are considered in Section 3. In Section 4, we concentrate upon the model under the real world measure and use a regime-switching Esscher transform to obtain the dynamics in the risk-neutral world.

2. Markov models and vanilla call options

In this section, we consider the pricing of a European call option when the price of the risky asset $\{S(t)\}$ is modelled under a risk-neutral measure $\mathbb{Q}$ by the exponential of a one-dimensional Markov-Modulated Lévy Process. Although the pricing formulae presented here are either not new or represent straightforward generalizations of some known results (see for example [1,6,7]), we present the derivation methods since we will use them later when dealing with multivariate options. Moreover, we will give some original regime-switching examples that can be treated in this Markovian framework.

To formulate the model we define the following processes:

- A Markov process $\{M(t) \mid t \in \mathbb{R}^+\}$ on the state space $\{1, 2, \ldots, N\}$ and determined by the generator $Q \in \mathbb{R}^{N\times N}$ and the initial vector $\boldsymbol{p} \in \mathbb{R}^N$,
- N independent Lévy processes $\{Y_j(t) \mid t \in \mathbb{R}^+\}\, j = 1, 2, \ldots, N$ determined under the measure $\mathbb{Q}$ by the characteristic functions:

$$\mathbb{E}_{\mathbb{Q}}\left[e^{iuY_j(t)}\right] = e^{-\Phi_j(u)t}, \quad u \in \mathbb{C}. \tag{2.1}$$

Based on these processes, we define the MMLP $\{X(t) \mid t \in \mathbb{R}^+\}$ as follows: $X(0) = 0$ and $\forall t > 0$:

$$dX(t) = \sum_{j=1}^{N} dY_j(t)\, \mathbb{1}_{M(t)=j}. \tag{2.2}$$

Intuitively, when the Markov process M is in state j, the process X behaves like the Lévy process Y_j.