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Weak Galerkin finite element methods for Sobolev equation

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Abstract. We present some numerical schemes based on the weak Galerkin finite element method for one class of Sobolev equations, in which differential operators are approximated by weak forms through the usual integration by parts. In particular, the numerical method allows the use of discontinuous finite element functions and arbitrary shape of element. The proposed schemes will be proved to have good numerical stability and high order accuracy when time variable is continuous. Also an optimal error estimate is obtained for the fully discrete scheme. Finally, some numerical results are given to verify our analysis for the scheme.

AMS subject classifications. 65M15, 65M60
Keywords. Sobolev equation, weak Galerkin, weak gradient, discrete weak gradient, error estimate

1 Introduction

Sobolev equations arise in the flow of fluids through fissured rock, thermodynamics and other applications. They are some important partial differential equations, which include a third order mixed derivative with respect to time and space, in practical use. They are used to describe wave motion in media with balancing with dispersion and diffusion, which are important not only in hydrodynamics but also in many other disciplines of engineering and science.

We consider the following Sobolev equation

$$\begin{aligned} &U_t - \nabla\cdot(\mu\nabla U_t + \varepsilon\nabla U) = f(x,t), && x\in\Omega, \quad t\in(0,T], \\ &U = \mathbb{u}(x), && x\in\Omega, \quad t=0, \end{aligned} \tag{1.1}$$

with homogenous Dirichlet boundary condition, where $U(x,t)$ is the unknown solution. The domain $\Omega\subset R^2$ is a bounded with boundary $\partial\Omega$. The $\mathbb{u}(x)$, $f(x,t)$ are sufficiently