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Two-dimensional wavelets collocation method for electromagnetic waves in dielectric media

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ABSTRACT

In this article, we deal with a numerical wavelet collocation method (NWCM) using a technique based on two-dimensional wavelets (TDWs) approximation proposed for the fractional partial differential equations (FPDEs) for electromagnetic waves in dielectric media (EWDM). By implementing the Riemann–Liouville fractional derivative, TDWs approximation and its operational matrix along with collocation method are utilized to reduce FPDEs firstly into weakly singular fractional partial integro-differential equations (FPIDEs) and then reduced weakly singular FPIDEs into system of algebraic equation. Using Legendre wavelet approximation (LWA) and Chebyshev wavelet approximation (CWA), we investigated the convergence analysis and error analysis of the proposed problem. Finally, some examples are included for demonstrating the efficiency of the proposed method via LWA and CWA respectively.

1. Introduction

Wavelets are powerful tool which have been used in numerical techniques. Nowadays, wavelets theory is mostly used in the field of applied science and engineering. Also, this allow the accurate representation of a variety of functions and operators. Recently, wavelets have been found their location in many applications (see for instant [1–3]). Particularly, wavelets are very successfully used in signal analysis [4]. It is proved that wavelets are powerful tool to explore new direction in solving partial differential equations. Wavelets are localized functions [5], which are the basis for energy-bounded functions [6] and in particular for $L^2(R)$. So, we implement orthogonal wavelet function in our proposed method. The most frequently used orthogonal function are Legendre function [7], Chebyshev [8], Laguerre polynomials [9], etc. The main notion of using an orthogonal basis is that the problem under consideration reduces to a system of linear or nonlinear algebraic equations. This can be done by truncated series of orthogonal basis function for the solution of the problem using the operational matrices (see for instant [10–12]). It is noted that wavelets operational matrix method not only simplifies the problem but also speedup the computation. Therefore, in the last two decades different families of wavelets have been widely used for solving FPDEs.

FPDEs have been one of the essential tools for various areas of applied Mathematics (see for instant [1–3]). FPDEs occur naturally in many fields of science and engineering. In recent years fractional derivatives have found numerous applications

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in many fields of physics, finance and hydrology [13]. Also, fractional analysis has established so many applications in recent studies in mechanics, and physical sciences phenomena in area like diffusion process [14], electrochemistry [15], arterial sciences [16], the theory of ultra-slow processes [17], etc.

Fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. A great deal of effort has been expanded over the last 15 years or so in attempting to find robust and stable numerical and analytical methods for solving FPDEs of physical interest.

In this paper, we present a NWCM by using two wavelets to solving FPDEs for EWDM (see [18]) as follows:

$$({}_0D_t^{\alpha}u)(t,x) - \lambda_1({}_0D_t^{\beta}u)(t,x) - \lambda_2\nabla^2 u(t,x) = f(t,x) \tag{1}$$

with initial condition

$$u(t,x) = 0, \quad \forall\, t \leqslant 0,\ u(t,x) \neq 0,\ \forall\, 0 < t < 1,\ 0 < x < 1$$

where the constant coefficients λ_1 and λ_2 depend on the frequency independent properties of medium and $1 \leq \beta < \alpha < 3$. Also, both the fractional derivatives present in Eq. (1) are defined in the Riemann–Liouville derivative sense. Eq. (1) can be considered as a generalization of the so called Szabo equation [19], which describes lossy propagation of acoustical waves in media with power law attenuation. Note that such a form allows simultaneous consideration of both regimes, before and after the peak frequency and the transition between them.

The rest of the paper are as follows: In Section 2 introduced preliminaries of Riemann–Liouville derivative for FPDEs. In Section 3, we discussed Legendre wavelet and their properties. In Section 4, we discussed Chebyshev wavelet and their properties. In Section 5, we defined function approximation. In Section 6, we constructed operational matrices of differentiation and integration. In Section 7, we discussed the method of solution of proposed problem. In Sections 8 and 9, we discussed the convergence analysis and error analysis respectively. In Section 10, we demonstrate the accuracy of the proposed method by several examples.

2. The fractional derivative in the Riemann–Liouville senses

In this section, we are recalling the necessaries of the calculus. The fractional calculus is a name for the theory of integrals and derivatives of arbitrary order, which unifies and generalizes the notions of integer-order differentiation and n-fold integration. So involving in our problem FPDEs solving by the Partial Riemann–Liouville fractional derivative with respect to x defined as follows (see [20]):

$$({}_0D_t^{\alpha}u)(t,x) = \frac{1}{\Gamma 1 - \{\alpha\}}\left(\frac{\partial}{\partial t}\right)^{[\alpha]+1}\int_0^t \frac{u(t,x)}{(t-s)^{\{\alpha\}}}ds \quad \forall\, t > 0,\ x > 0,\ \alpha > 0$$

and

$$({}_0D_t^{\beta}u)(t,x) = \frac{1}{\Gamma 1 - \{\beta\}}\left(\frac{\partial}{\partial t}\right)^{[\beta]+1}\int_0^t \frac{u(t,x)}{(t-s)^{\{\beta\}}}ds \quad \forall\, t > 0,\ x > 0,\ \beta > 0$$

where, $[\alpha]$, $[\beta]$ and $\{\alpha\}$, $\{\beta\}$ being the integral and fractional parts of α and β respectively. This reduced the FPDEs into FPIDEs and this provide solvability of FPDEs in easy way.

3. Legendre wavelet and their properties

The well-known Legendre polynomials are defined on the interval $[-1, 1]$ and can be determined with the aid of the following recurrence formulae:

$$(m+1)L_m(y) = (2m+1)yL_m(y) - mL_{m-1}(y), \quad m \in N, \tag{2}$$

where,

$$L_0(y) = 1, \qquad L_1(y) = y.$$

In order to use Legendre polynomials on the interval $[0, 1]$ we define the so-called shifted Legendre polynomials by introducing the change of variable $y = 2x - 1$. Let the shifted Legendre polynomials $L_m(2x-1)$ be denoted by $P_m(x)$. Then $P_m(x)$ can be obtained as follows:

$$(m+1)P_{m+1}(x) = (2m+1)(2x-1)P_m(x) - mP_{m-1}(x), \quad m = 1, 2, 3, \ldots,$$

where, $P_0(x) = 1$ and $P_1(x) = 2x - 1$. The shifted Legendre polynomial $P_m(x)$ has the following analytic form:

$$P_m(x) = \sum_{k=0} m(-1)^{m+k}\frac{(m+k)!x^k}{(m-k)!(k!)^2}$$

and the orthogonality condition as follows:

$$\int_0^1 P_m(x)P_n(x)dx = \begin{cases} \dfrac{1}{2m+1} & \text{if } m = n; \\ 0 & \text{if } m \neq n. \end{cases} \tag{3}$$