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A Framework for the Upscaling of the Electrical Conductivity in the Quasi-static Maxwell's Equations

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Abstract

Electromagnetic simulations of complex geologic settings are computationally expensive. One reason for this is the fact that a fine mesh is required to accurately discretize the electrical conductivity model of a given setting. This conductivity model may vary over several orders of magnitude and these variations can occur over a large range of length scales. Using a very fine mesh for the discretization of this setting leads to the necessity to solve a large system of equations that is often difficult to deal with. To keep the simulations computationally tractable, coarse meshes are often employed for the discretization of the model. Such coarse meshes typically fail to capture the fine-scale variations in the conductivity model resulting in inaccuracies in the predicted data. In this work, we introduce a framework for constructing a coarse-mesh or upscaled conductivity model based on a prescribed fine-mesh model. Rather than using analytical expressions, we opt to pose upscaling as a parameter estimation problem. By solving an optimization problem, we obtain a coarse-mesh conductivity model. The optimization criterion can be tailored to the survey setting in order to produce coarse models that accurately reproduce the predicted data generated on the fine mesh. This allows us to upscale arbitrary conductivity structures, as well as to better understand the meaning of the upscaled quantity. We use 1D and 3D examples to demonstrate that the proposed framework is able to emulate the behavior of the heterogeneity in the fine-mesh conductivity model, and to produce an accurate description of the desired predicted data obtained by using a coarse mesh in the simulation process.

Keywords: Numerical Homogenization, Maxwell's Equations, Simulation, Electrical Conductivity, Finite Volume, Geophysical Electromagnetic Methods
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1. Introduction

Forward modeling of quasi-static Electromagnetic (EM) responses — fields and fluxes — significantly enhances our understanding of how variations in electrical conductivity impact EM data [1, 2]. As a consequence, tools to simulate EM responses have been extensively used in a variety of scenarios, including medical applications [3], and geophysical applications such as mineral and hydrocarbon exploration, groundwater monitoring, and geotechnical and environmental investigations (cf. [4, 5, 6]). Our particular interest is in the simulation of quasi-static EM responses over highly heterogeneous geologic settings, which are typically computationally expensive problems [7, 2]. Robust and accurate simulations of such settings require very fine meshes that are difficult, if not impossible, to work with as they translate into solving huge systems of equations (often on the order of tens of millions or even billions of unknowns). The simulation's cost is mainly due to the fact that the mesh must accurately capture the conductivity structures, which vary

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