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BIORTHOGONAL MULTIWAVELETS ON THE INTERVAL FOR NUMERICAL SOLUTIONS OF BURGERS' EQUATION

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ABSTRACT. To numerically solve the Burgers' equation, in this paper we propose a general method for constructing wavelet bases on the interval $[0,1]$ derived from symmetric biorthogonal multiwavelets on the real line. In particular, we obtain wavelet bases with simple structures on the interval $[0,1]$ from the Hermite cubic splines. In comparison with all other known constructed wavelets on the interval $[0,1]$, our constructed wavelet bases on the interval $[0,1]$ from the Hermite cubic splines not only have good approximation and symmetry properties with extremely short supports, but also employ a minimum number of boundary wavelets with a very simple structure. These desirable properties make them to be of particular interest in numerical algorithms. Our constructed wavelet bases on the interval $[0,1]$ are then used to solve the nonlinear Burgers' equation. Our method is based on the finite difference formula combined with the collocation method. Therefore, our proposed numerical scheme in this paper is abbreviated as MFDCM (Mixed Finite Difference and Collocation Method). Some numerical examples are provided to demonstrate the validity and applicability of our proposed method which can be easily implemented to produce a desired accuracy.

1. INTRODUCTION

1.1. **Motivation.** Wavelet bases have been successfully applied for the numerical solutions of PDEs (see [9, 13, 19] and references therein). Many applications require wavelet bases on a finite interval rather than on the whole real line. Examples of such situations can be found in numerical analysis for differential equations with certain boundary conditions, and in image processing whose domains are the Cartesian products of finite intervals. The construction of wavelet bases on finite intervals has been extensively discussed in the literature and several approaches have been developed to adapt wavelets on the real line $\mathbb{R}$ to the interval $[0,1]$. Compactly supported smooth orthogonal wavelets were constructed by Daubechies in [22] and compactly supported biorthogonal wavelets were constructed by Cohen et al. in [15]. Such Daubechies orthogonal wavelets were adapted to construct wavelets on the interval $[0,1]$ by Meyer in [50] and by Cohen et al. in [14]. Semi-orthogonal wavelets were constructed by Chui and Wang in [12] and such spline wavelets were adapted to the interval $[0,1]$ by Chui and Quak in [11]. The construction of compactly supported biorthogonal multiwavelets have been studied in [64, 65]. Biorthogonal spline wavelets on the interval have been studied by Dahmen et al. in [21] and are derived from biorthogonal wavelets on the real line using B-splines. Heil et al. in [32] considered the construction of multiwavelets on the real line from Hermite cubic splines. The widely used Hermite cubic splines $\phi = (\phi^1, \phi^2)^\mathsf{T}$ are given by

$$\begin{aligned}\phi^1(x) &:= (1-3x^2-2x^3)\chi_{(-1,0]} + (1-3x^2+2x^3)\chi_{(0,1]},\\ \phi^2(x) &:= (x+2x^2+x^3)\chi_{(-1,0]} + (x-2x^2+x^3)\chi_{(0,1]}.\end{aligned} \tag{1.1}$$

Note that $\phi \in (C^1(\mathbb{R}))^2$ is a Hermite interpolant of order 1 satisfying

$$\phi^1(k) = \boldsymbol{\delta}(k), \qquad [\phi^1]'(k) = 0, \qquad \phi^2(k) = 0, \qquad [\phi^2]'(k) = \boldsymbol{\delta}(k) \qquad \forall k \in \mathbb{Z}, \tag{1.2}$$

Key words and phrases. Hermite cubic splines; Biorthogonal multiwavelets with symmetry; Burgers' equation; Collocation method; Operational matrix of derivative; Operational matrix of product.

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