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Accurate solution of the Thomas-Fermi equation using the fractional order of rational Chebyshev functions

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Abstract

In this paper, the nonlinear singular Thomas-Fermi differential equation for neutral atoms is solved using the fractional order of rational Chebyshev orthogonal functions (FRCs) of the first kind, $FT_n^{\alpha}(t, L)$, on a semi-infinite domain, that L is an arbitrary numerical parameter. First, using the quasilinearization method, the equation be converted into a sequence of linear ordinary differential equations (LDEs), and then these LDEs are solved using the FRCs collocation method. Using 300 collocation points, we have obtained a very good approximation solution and the value of the initial slope $y'(0) = -1.588071022611375312718684509423950109 5$, highly accurate to 37 decimal places.

Keywords: Thomas-Fermi equation; Fractional order of rational Chebyshev functions; Quasilinearization method; Collocation method; Nonlinear ODE; Semi-infinite domain.
2010 MSC: 34B16, 34B40, 74S25

1. Introduction

In this section, we expressed the introduction of methods used to solve problems in an infinite or semi-infinite domain. Also, we have tried that provide a proper history for Thomas-Fermi equation and show the progress of this problem.

1.1. Differential equations on infinite and semi-infinite domain

Many problems arising in fluid dynamics, quantum mechanics, astrophysics, and other fields are defined on infinite or semi-infinite domains. There are different approaches to solve this type of equation, such as numerical and semi-analytical methods.

1. **Numerical methods:** Different numerical methods have been introduced for solving problems on various domains such as finite difference method